Structural Change in Production Networks and Economic Growth*

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Abstract: We study structural change in production networks for intermediate inputs (input-output network) and new capital (investment network). For each network, we document that the share of output produced by services (relative to goods) is rising over time. While the relative prices of services that produce intermediates and consumption are rising, we find that the relative price of services that produce investment is falling over time. We then develop a multi-sector growth model to study these trends and their implications for economic growth. To match the relative price trends, inputs to intermediates production are complements and inputs to investment production are substitutes. Hence, structural change endogenously reallocates resources to the slowest growing intermediates producers and the fastest growing investment producers. Growth accounting exercises reveal that investment-specific technical change accounts for an increasing share of U.S. aggregate growth, with 20% of aggregate growth since 2000 due to investment structural change. Growth projections from our model show that structural change within investment networks alone can offset stagnating or declining growth in other sectors due to Baumol's cost disease.

JEL: E23, O14, O40, O41

Keywords: structural change, input-output network, investment network, economic growth, technological change, balanced growth

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1. Introduction

Production networks—the distribution of transactions between sectors that produce and purchase commodities used in production—play a central role in shaping economic fluctuations and growth.[1] However, with changes in technology and the organization of production, these networks evolve over time. This paper documents structural change in the production networks for investment and intermediates and shows how these changes influence economic growth.

Our analysis proceeds in three steps. First, we document a rising share of output produced by services (e.g., financial services, professional/technical services) relative to goods (e.g., manufacturing, construction) within both the intermediates and investment network and then construct separate price series for intermediates and investment (and consumption) produced by goods and services sectors. Second, we develop a multi-sector model with production networks for intermediate inputs and investment to understand the relationship between structural change and economic growth. Finally, we solve a transition path of the model, calibrated to match observed patterns of structural change, and analyze the importance of structural change for historical U.S. growth from 1947-2020 and projected future growth in GDP per worker.

The main contributions of our paper follow from our measurement of goods and services prices for different uses of output—consumption, investment, and intermediates. While the price of services relative to goods used as consumption (e.g., education and health care) and intermediate inputs (e.g., financial services and wholesale trade) are continually rising, the relative price of services used as investment (e.g., software development and R&D) is actually falling. Within our calibrated model, our finding of opposing price trends in intermediates and investment networks implies that goods and services are substitutes in the production of investment and complements in the production of intermediates and consumption. Thus, structural change in the investment network means that expenditures reallocate toward the producers with the fastest total factor productivity (TFP) growth, endogenously increasing economic growth—the "frontiers" of growth. Likewise, complementarity between goods and services in intermediates (and consumption) slows

[1] Among many possible examples, see Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Baqaee and Farhi (2019), Foerster, Hornstein, Sarte and Watson (2019), Kopytov, Mishra, Nimark and Taschereau-Dumouchel (2023), vom Lehn and Winberry (2022).

growth as structural change reallocates expenditures to producers with the slowest TFP growth—the "bottlenecks" of growth. As a consequence, the net impact on aggregate growth is a quantitative question. Projections generated from our calibrated model suggest that continuing structural change in the investment network is likely sufficient to offset concerns about declining growth due to Baumol's (1967) cost disease.

The first step of our analysis uses national accounting data to document changes in the producers of intermediates and investment. We find that goods sectors systematically produce a smaller share of output in each production network over time, offset by increased production by service sectors. However, the specific goods and services sectors in which production is rising and falling vary by network. These changes are large, similar in magnitude to structural change observed in consumption expenditures, and widespread, occurring in nearly all sectors and in many countries.

Given that the goods and services sectors producing consumption, investment, and intermediates are distinct, we measure the price of goods and services separately for each of these final uses. To do so, we leverage expenditure-side national accounting data, mapping prices from the Bureau of Economic Analysis' (BEA) final expenditure data on 68 consumption and 30 investment commodities in the National Income and Product Accounts (NIPA) to the goods or services sectors that produce these commodities. For intermediates, which aren't included in GDP and are therefore not in NIPA expenditure data, we infer prices as residuals from the BEA's sectoral gross output prices, which are a weighted average of consumption, intermediates, and investment prices. Not only is this approach internally consistent with published aggregated price data, it also produces intermediates price series that align almost perfectly with the producer price indices (PPI) published by the U.S. Bureau of Labor Statistics (BLS) for the select years that they are available.

This measurement approach yields distinct relative price patterns for goods and services across consumption, investment, and intermediates. Recent work allowing for structural change in investment (e.g., Herrendorf, Rogerson and Valentinyi, 2021; García-Santana, Pijoan-Mas and Villacorta, 2021; Sposi, Yi and Zhang, 2021) measures a single price for goods and services across uses and finds that the price of services has been rising faster than that of goods. We find a similar result for consumption and intermediates, but the opposite result for investment, with the price of services declining relative to goods. The key to our result is aggregation bias in the sectoral gross output prices typically used to measure goods and services prices. Since only around 5% of services output is used for investment, the price of investment produced by services sectors is not accurately represented in the aggregate price index for the services sector as a whole. We show that this aggregation bias persists even when using more disaggregated sectoral output prices.

In our second step, we extend the multi-sector neoclassical growth model to allow for production networks in intermediates and investment to analyze how these patterns in relative expenditures and prices impact aggregate growth. In our model, each sector produces gross output using a combination of capital, labor, and intermediate inputs. Production networks are modeled as each sector's intermediates and investment being a constant elasticity of substitution (CES) bundle of inputs, purchased from many different sectors. This setup implies that changes in relative prices across sectors, induced by changes in technology, can generate changes in the composition of intermediates and investment production—i.e., structural change in production networks. To highlight the implications of structural change for growth, we consider a special case of the model that admits a balanced growth path. We show that the higher the elasticity of substitution, the faster growth will be in either network and that the composition of aggregate growth endogenously shifts towards the network with the higher elasticity of substitution.

Finally, to study the quantitative implications of structural change in production networks, we solve a calibrated transition path of our model and consider counterfactual exercises where we vary which TFP series are fed into the model and/or the values for the elasticities of substitution. Our baseline calibration studies six sectors, with each of the consumption, investment, and intermediates sectors split into a goods and services subsector—a natural extension of Greenwood, Hercowitz and Krusell (1997) and Ngai and Samaniego (2009). We calibrate the elasticities of substitution in the model to obtain the best fit to observed patterns in expenditure shares and relative prices. Our calibration implies that goods and services are complements within consumption and intermediates, but substitutes within investment.

We consider two types of counterfactual exercises: historical counterfactuals that decompose the sources of U.S. growth in GDP per worker from 1947-2020 and growth projections for future decades assuming constant sectoral TFP growth. Consistent with the model's theoretical implications, TFP growth of investment producers is increasingly important over time, with its contribution to aggregate growth rising from roughly 40% in the 1960s to more than 75% since 2000. Furthermore, roughly 20% of aggregate growth since 2000 is driven by structural change in the investment network alone. In contrast, we find that intermediates-specific technical change has stagnated, contributing negatively to growth after 2000, and hence playing a large role in the recent growth slowdown. To analyze fears of slowing long-run growth due to structural change and Baumol's cost disease, we use the model to project future growth. Despite recent sluggish growth, our projections do not imply a further slowdown. If anything, our model predicts mild acceleration, as structural change within the investment network is sufficient to offset the drag of Baumol's cost disease due to structural change in consumption and intermediates.

Related Literature Our paper connects two large strands of literature: the study of production networks as propagation mechanisms for economic fluctuations and growth and the study of long-run structural transformation from goods to services. The production networks literature primarily emphasizes the role of static production networks in shaping fluctuations. The literature on structural change typically focuses on multi-sector models that either abstract from production networks (or treat them implicitly in a "value added" specification) or do not allow these networks to change over time.[2] Our contribution lies in studying the intersection of these two phenomena.

Several studies have explored how production networks shape long-run growth (e.g., Ngai and Samaniego, 2009; Moro, 2015; Foerster et al., 2019; Valentinyi, 2021). Particularly relevant is Ngai and Samaniego (2009), who examine how technological change in intermediate goods sectors affects the composition of growth through investment-specific technological change (as in Greenwood et al., 1997). Similarly, Foerster et al. (2019) investigate how production networks, especially investment networks, influence which sectors' TFP growth accounts for aggregate growth slowdowns. Our approach extends these insights by analyzing growth patterns across sectors defined both by their product market and the use of those products and allowing production networks

[2] For example, Herrendorf, Rogerson and Valentinyi (2013) introduce value-added measures of structural change that implicitly embed indirect contributions to final production occurring via the input-output network. Absent using this approach, Herrendorf et al. (2013) and Herrendorf, Rogerson and Valentinyi (2014) argue that using expenditure-side prices to calibrate models of structural change is only appropriate when the model accounts for the input-output structure of the economy. We take this alternate approach in this paper. Beyond our new result on the substitutability between goods and services in investment, explicitly modeling the input-output network is important as we document that structural change in the network of intermediates production accounts for nearly half of the measured structural change in consumption and investment.

to change endogenously, which introduces the possibility of Baumol's cost disease.

Our work also builds on the literature that disaggregates the services sector in different ways (Buera and Kaboski, 2012; Duarte and Restuccia, 2020; Eckert et al., 2019; Buera, Kaboski, Rogerson and Vizcaino, 2022; Duernecker, Herrendorf and Valentinyi, 2024). Our approach distinguishes services subsectors by the use of their products (consumption, investment, intermediates) and emphasizes the elasticity of substitution between goods and services in producing each final use. Since these sectors aren't directly identifiable using standard classifications, we use expenditure-side data to measure prices and productivity, similar to the measurement of investment-specific technical change in Greenwood et al. (1997).

Finally, our work relates to recent literature that has observed and begun to analyze structural change in production networks. Early work by Berlingieri (2013) and Galesi and Rachedi (2018) provides some evidence that services sectors have been rising in importance for intermediates production and more recent work by Sposi (2019) and Sposi et al. (2021) has incorporated intermediates structural change to their models to explain the hump-shaped rise and fall of manufacturing and the global distribution of manufacturing. Most closely related to our work, Herrendorf et al. (2021) and García-Santana et al. (2021) extend structural change models to incorporate structural change in investment. Our work builds on these analyses by providing an explicit quantitative analysis of structural change in intermediates and allowing for heterogeneous prices of goods and services by use, allowing us to discover the substitutability of goods and services inputs in investment.

2. Structural Change in Production Networks

We focus on structural change within two production networks: the input-output network (the sectoral distribution over the production and purchases of intermediate inputs) and the investment network (the distribution over the production and purchases of new capital). The primary data sources for measuring these production networks in the U.S. are the Make and Use Tables from the U.S. Bureau of Economic Analysis' (BEA) Input Output Database. These tables contain data for each sector on the value of gross output, value-added, intermediate input purchases (from every sector), and final uses (consumption, investment, etc.) of each sector's production. The database begins in 1947, and we study patterns of change through 2020. For measuring the investment network, we also use data from vom Lehn and Winberry (2022), who combine BEA Input Output data with BEA Fixed Assets tables to construct a time series of the investment network that covers a similar time frame and level of sector detail; we extend their data to run through 2020.[3] Additional details regarding data sources and measurement are available in Appendix A.

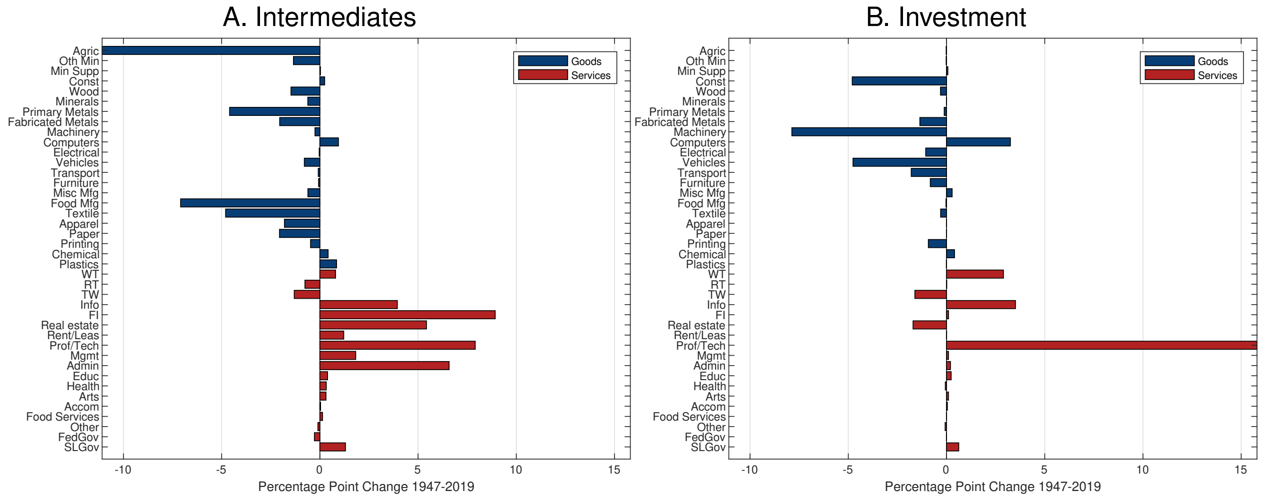
These sources allow us to compile a dataset that provides consistent coverage of U.S. production networks for 40 NAICS-defined sectors, including agriculture and government.[4] In our main analyses, we define "goods" sectors as all agriculture, mining, construction, and manufacturing sectors (22 in total) and "services" as all remaining sectors (18 in total). We exclude three energy-intensive sectors—oil and gas extraction, utilities, and petroleum and coal manufacturing— whose outcomes are distinct from other sectors because of short- and long-run volatility in energy markets. However, as we show in Appendix Appendix B, these sectors play no role in structural change in production networks. Since price growth in these sectors can be so large as to overshadow price growth in other producers of intermediate inputs, we exclude them from our analysis.

The two production networks are defined as a pair of matrices for each year t, where element (i, j) of each matrix reports expenditures by sector j on intermediates or investment produced by sector i in year t. As we show in Appendix A, these networks are generally sparse; for any given sector, the majority of investment and intermediates are purchased from a small set of sectors. The distribution of investment producers is fairly similar across sectors, as most sectors purchase investment goods from a collection of prominent investment hubs—construction (structures), machinery and motor vehicles manufacturing (equipment), and information and professional and technical services (intellectual property). However, there is much more sector-specificity as to which sectors are important suppliers of intermediates. In particular, the input-output network features significant homophily—goods sectors play a large role as intermediates suppliers for goods sectors and

[3]These data include imported intermediates, consumption, and investment, reflecting the full set of intermediate and investment purchases by each sector, regardless of geographic origin. Because the BEA only reports total spending on imports and doesn't differentiate how these imports are used, we cannot remove imports from the data within our measurement framework. That said, throughout the period 1947-2020, imports are never more than 8% of total consumption, intermediates, and investment in any given year, so changes in imports cannot account for the quantitatively large aggregate patterns of structural change we document below.

[4]Table A.1 in Appendix B lists each of the 40 sectors and their corresponding NAICS codes. More recent vintages of the Input Output database have greater sectoral detail, but given our interest in structural change over the long run, we focus on these 40 sectors, which can be observed throughout the period 1947-2020.

Figure 1: Changes in Production Share of Intermediates and Investment: 1947-2019



Notes: Each bar represents the change in the share of intermediates (panel A) or investment (panel B) produced by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

services sectors play a large role as intermediates suppliers for services sectors.

To measure structural change in these production networks, we analyze changes in the fraction of aggregate intermediates or investment spending on output produced by sector i. For any given sector j, we define the share of intermediates and investment purchased from sector i as s_{ijt}^M and s_{ijt}^X, respectively.[5] The share of aggregate intermediates or investment produced by sector i, s_{it}^M and s_{it}^X, respectively, can be expressed as a weighted average of the within-sector shares, s_{ijt}^M and s_{ijt}^X:

$$s_{it}^M = \sum_j \frac{P_{jt}^M M_{jt}}{\sum_k P_{kt}^M M_{kt}} s_{ijt}^M \quad \text{and} \quad s_{it}^X = \sum_j \frac{P_{jt}^X X_{jt}}{\sum_k P_{kt}^X X_{kt}} s_{ijt}^X, \tag{1}$$

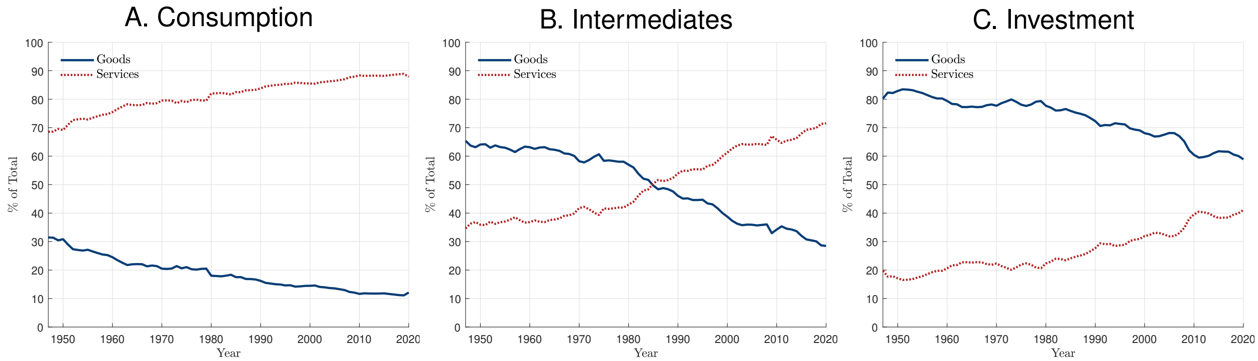
where $P_{jt}^M M_{jt} \equiv \sum_i P_{it} M_{ijt}$ is the total spending on intermediates by sector j (expressions for investment are defined analogously). Notice that changes in s_{it}^M and s_{it}^X reflect both changes within sectors and changes in the composition of spending between sectors.

Figure 1 plots long differences in s_{it}^M and s_{it}^X for each sector i from 1947 to 2019.[6] Changes in these production networks are broadly characterized by a shift from goods sectors (blue bars)

[5]Formally, $s_{ijt}^M = \frac{P_{it} M_{ijt}}{\sum_l P_{lt} M_{ljt}}$, where P_{it} represents the price of sector i's intermediates at time t and M_{ijt} represents the quantity of intermediate inputs purchased from sector i by sector j, with an analogous expression for investment. We use information on total payments from sector j to sector i in year t, $P_{it} M_{ijt}$, as price and quantity are not separately observed in the network data; this notation allows for easy comparison to our model.

[6]We end in 2019 to avoid any unusual endpoint effects with the onset of the COVID-19 pandemic.

Figure 2: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, 1947-2020



Notes: The figures plot the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue, solid line) and the services sector (red, dotted line). Sectors included in goods and services can be seen in Table A.1.

to services sectors (red bars). However, the specific sectors with the largest changes differ across intermediates and investment producing sectors. For intermediates, the largest increases occurred in information services, finance/insurance, real estate, professional/technical services, and administrative and waste services; the largest declines were in agriculture, primary metals, food and beverage manufacturing, and textile manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest declines were in machinery, construction, and motor vehicle manufacturing.[7]

Given the general sectoral patterns in Figure 1, Figure 2 plots time series of s_{it}^M and s_{it}^X at the two sector level (goods and services). As structural change in consumption appears in our quantitative analysis, Figure 2 also plots the goods and services shares of total consumption production. This figure summarizes the main aggregate stylized facts about changing production networks: for each of the three aggregate uses of sectoral output, the fraction produced by services sectors is increasing over time, with the services share of investment and consumption rising by roughly 20 percentage points and the services share of intermediates increasing by more than 35 percentage points.

The rising services share in each production network may reflect increased spending on service inputs within sectors or increased intermediates spending by sectors that use services intermediates more intensively. A shift-share decomposition (reported in Appendix B) reveals that within-sector changes account for roughly 50% of the increase in services-produced intermediates, compared

[7] The time series of production shares for these sectors are shown in Appendix B.

to 75-100% for services-produced investment. These findings reflect the higher sector-specificity within the intermediates network compared to the investment network and account for the fact that the intermediates network experienced more pronounced structural change than either consumption or investment.

These stylized facts relate to the "value-added" measurement approach introduced by Herrendorf et al. (2013). Their approach implicitly includes structural change in the intermediates network by using input-output data to identify each sector's overall contribution to producing consumption or investment. Given the quantitatively large changes within the intermediates network (Figure 2), we measure what portion of value-added structural change can be attributed to the intermediates network in Appendix B. Our decomposition shows that structural change in the intermediates network alone accounts for roughly 50% of structural change in consumption and investment value-added.

Finally, in Appendix B, we provide two additional sets of results highlighting the robustness of the stylized facts presented here. First, we provide evidence that structural change in intermediates is not merely outsourcing of services tasks. Second, we show that structural change in production networks is observed within almost all sectors in the U.S. and that these patterns are also observed in many other high-income countries, using data from the World Input Output Database (WIOD: Timmer, Dietzenbacher, Los, Stehrer and de Vries, 2015; Woltjer, Gouma and Timmer, 2021).

3. The Relative Price of Services for Consumption, Investment, and Intermediates

Going back at least to Baumol (1967), a commonly proposed explanation for structural change patterns as illustrated in Figure 2 are movements in relative productivity, which in turn drive changes in relative prices. The predominant measurement approach in the existing literature assumes a single price for goods and a single price for services, aggregated across standard sectoral definitions (e.g., Herrendorf et al., 2021; García-Santana et al., 2021; Sposi et al., 2021). While parsimonious, this approach masks important heterogeneity in price trends across sectors specializing in the production of consumption, investment, and intermediates. For example, in the previous section, we observed that the underlying sectors contributing to the rise of services and the fall of goods are different across investment and intermediates.[8] Thus, to capture this heterogeneity, we instead compute goods and services prices separately for each of the use categories of consumption, investment, and intermediates.

A key challenge in measuring goods and services prices by use is that standard data sources only provide detailed information for either sectors or uses, but not both simultaneously. For instance, the BEA Industry accounts offer detailed gross output prices by sector, clearly distinguishing goods from services, but don't differentiate these sectoral prices by the use of each sector's output.[9] Consequently, the ability to construct use-specific prices with gross output data is limited by the extent to which goods and services sub-sectors specialize in producing consumption, intermediates, or investment that can be aggregated into a single price by use. However, as we demonstrate, there is a limited degree of specialization across our 40 consistently defined sectors, which significantly constrains measured price variation across uses. This diminishes the viability of a measurement approach that relies exclusively on gross output prices from the BEA Industry Accounts.

3.1. Price Measurement

To measure prices separately by use we instead draw on data for detailed consumption and investment commodities from the U.S. NIPA and then use Input Output data to map the production of these commodities to goods and services sectors. Expenditure-side NIPA data consistently cover expenditures and prices for 68 consumption and 30 investment commodities over our entire sample, 1947-2020. Examples of consumption commodities at this level of detail include household appliances, jewelry and watches, children's and infants' clothing, dental services, and purchased meals and beverages. Examples of investment commodities include non-mining structures, office and accounting equipment, construction machinery, software, and R&D investment. As emphasized by Herrendorf et al. (2013) and Herrendorf et al. (2014), these prices partially reflect the price of intermediate inputs used in the production of these commodities. Thus, they will only be consistent with a general equilibrium framework that explicitly models the input-output structure

[8] These differences are even more pronounced for consumption, where health services account for a big part of the rise of services, and produce virtually no intermediates or investment (see Table A.1).

[9] In its GDP by Industry database and integrated productivity accounts, the BEA publishes the price of the intermediates bundle each sector purchases. However, this bundle price does not identify the price of intermediate inputs produced by different sectors.

of the economy, which we develop in the next section.

Using expenditure-side price data ensures that we properly identify differences in prices by consumption and investment uses, but we must still map these prices into the NAICS-defined categories of goods and services. To map the prices of these NIPA commodities into our sectors, we use a combination of BEA Input Output data published in "bridge files" and "make tables"; these files have been used in other work (e.g., Bils, Klenow and Malin, 2013; vom Lehn and Winberry, 2022; Bergman, Jaimovich and Saporta-Eksten, 2023) to map commodities reported in NIPA to NAICS sectors. The combination of bridge and make files "bridges" the income and expenditure sides of national accounts, mapping NIPA commodities into the NAICS sectors that "make" them. Formally, the bridge and make files generate weights ξ^u_{jlt}, which make it possible to aggregate spending on individual commodities, l, for use u, at time t into the production of that use by sector j at time t. For example, for consumption, the value of consumption produced by sector j, $P_{jt}C_{jt}$ is given by a weighted sum of consumption spending on all commodities l: $P_{jt}C_{jt} = \sum_l \xi^C_{jlt} P^C_{lt} Q^C_{lt}$.

For example, consider the NIPA consumption commodity of new motor vehicles, on which 167 billion dollars were spent in 1997. The combination of bridge and make files provides a detailed breakdown of how much each NAICS sector contributed to the final value of new motor vehicles: 71% from motor vehicle manufacturing, 22% from retail trade, 3% from wholesale trade, 1% from transportation and warehousing services, and the remaining 2% from a handful of other sectors, such as fabricated metals and machinery.[10]

Based on these bridge and make files, we measure separate prices for consumption and investment produced by goods and services sectors using a Tornqvist index, which is a weighted average of price growth across consumption or investment commodities. For example, we measure the price for goods consumption, $P_{g-c,t}$, using NIPA price and spending data on 68 consumption com-

[10]NAICS-coded bridge files for consumption are only available starting in 1997. We thus assume that the bridge file entries for consumption prior to 1997 are the same as the 1997 data. However, the BEA does publish SIC-coded consumption bridge files going back to 1967, with less detail than the latest bridge files. We have explored converting these to NAICS codes and found only very minor changes in the bridge files between 1967 and 1997, supporting our use of the 1997 bridge files for earlier years. For investment, we use the bridge files from vom Lehn and Winberry (2022), who extend these files back to 1947.

modities (P^C_{lt} and $P^C_{lt}Q^C_{lt}$ for $l \in \{1, ..., 68\}$) according to the formula:

$$\Delta \ln(P_{g-c,t}) = \sum_{l=1}^{68} \mu_{glct} \Delta \ln(P^C_{lt}), \tag{2}$$

where $\mu_{glct} \equiv \frac{\xi^C_{glt} P^C_{lt} Q^C_{lt}}{\sum_{m=1}^{M} \xi^C_{gmt} P^C_{mt} Q^C_{mt}}$ is the weight (averaged across years $t-1$ and t) each consumption commodity receives in the goods consumption price index, with $P^C_{lt}Q^C_{lt}$ representing total spending on consumption commodity l and ξ^C_{glt} the fraction of consumption commodity l produced by goods sectors according to the BEA bridge/make files.[11] Calculations for the price of goods investment, services consumption, and services investment are analogous.

Finally, since intermediate inputs are excluded from GDP, their prices cannot be measured using expenditure-side NIPA data. However, gross output prices in the BEA Industry accounts represent an average of consumption, investment, and intermediates produced by each sector. Therefore, we can identify intermediate input prices for goods and services as the residual in gross output prices after removing consumption and investment prices. This approach ensures that our measured prices are fully consistent with the aggregated prices of the goods and services sectors.

For example, treating gross output prices as a Tornqvist index, we naturally measure price growth in intermediates produced by goods sectors as the residual of this index:

$$\Delta \ln(P_{g-m,t}) = \frac{1}{1 - \zeta^C_{gt} - \zeta^X_{gt}} \left(\Delta \ln P^{GO}_{gt} - \zeta^C_{gt} \Delta \ln P_{g-c,t} - \zeta^X_{gt} \Delta \ln P_{g-x,t} \right), \tag{3}$$

where ζ^i_{gt} is the average share (between $t-1$ and t) of goods gross output used for $i \in \{C, X\}$, and $\Delta \ln P^{GO}_{gt}$ is the log change in the goods sector gross output price.[12] The services-intermediates price is measured analogously.[13]

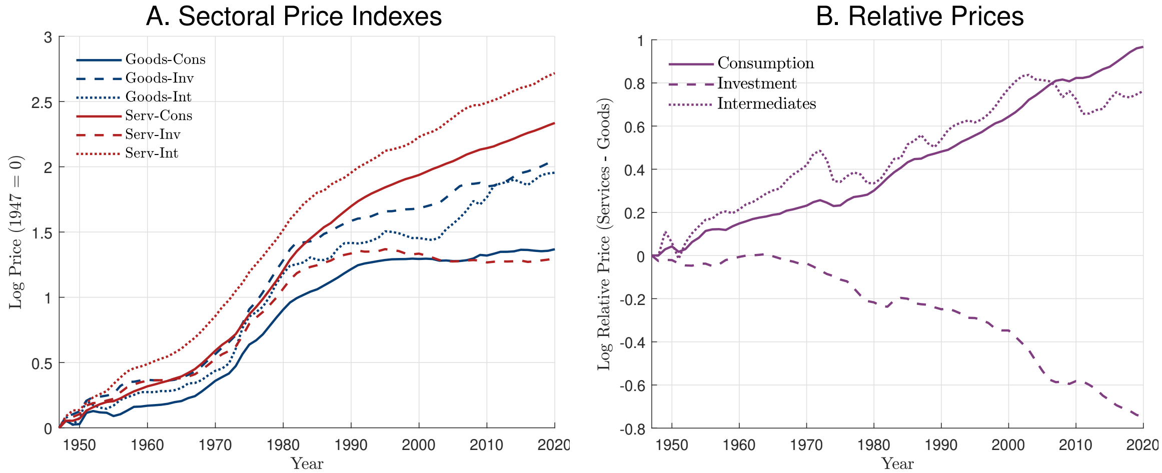
Figure 3 plots our price series for consumption, investment, and intermediates produced by goods and services sectors; panel A illustrates prices in log levels while panel B shows the log of

[11] In the case of investment, for consistency with how we measure structural change, we use the adjusted (for used goods, etc.) spending data by commodity used to construct the investment network data in vom Lehn and Winberry (2022). Using the raw NIPA data generates nearly identical results.

[12] We adjust gross output prices (and to a lesser extent consumption and investment prices) for oil/energy price spillovers before performing this procedure. The qualitative patterns of relative price movements across goods and services sectors are robust to not making these corrections (see Appendix C).

[13] Alternatively, the Producer Price Index (PPI) published by the U.S. Bureau of Labor Statistics (BLS) provides purchasers' prices for intermediate inputs produced by different sectors. These data provide a direct measure of intermediate input prices but have incomplete coverage of services sectors (roughly 85% of the services sectors producing intermediates) and prices for services intermediates are only available starting in 2009. Our intermediates prices are nearly identical to the published PPI data for prices of both goods and services intermediates (see Appendix C).

Figure 3: Prices of Goods and Services Consumption, Investment and Intermediates, 1947-2020



Notes: Panel A shows the time series of prices for each use (consumption, investment, or intermediates) produced by each sector (goods or services). Panel B shows the log of the price of services divided by the price of goods for each use. All series logged and normalized to 0 in 1947.

the price of services divided by the price of goods for each use. Consistent with the well-known price patterns in goods and services as a whole, the price of services relative to goods rises significantly over time for consumption and intermediates. Given the rising share of services expenditures for consumption and intermediates, rising relative prices are consistent with complementarity between goods and services inputs.

Surprisingly, and in contrast to aggregate price trends, the relative price of services investment is falling. This finding arises as investment inputs produced by services sectors are primarily information technology and intellectual property products (e.g., software and R&D) whose price has fallen significantly relative to the price of goods investment (e.g., equipment or structures). Given rising expenditures on services investment, falling relative prices are consistent with the idea that goods and services inputs to investment are *substitutes*. Hence, the economic consequences of structural change in investment may be significantly different than for consumption and intermediates.

In Appendix C, we show that this finding is robust to a variety of alternative specifications: aggregating investment prices with user cost weights instead of investment expenditures as recommended by Holden, Gourio and Rognlie (2020); focusing exclusively on equipment and software; making quality adjustments as in Cummins and Violante (2002); and holding the values of bridge files constant across all time periods. In all cases, the price of investment by services sectors is declining relative to the price of investment produced by goods sectors.

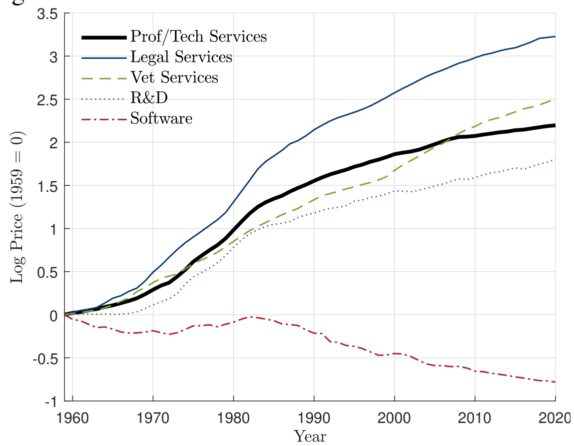
3.2. Aggregation Bias

The fact that relative price movements in goods and services investment are more consistent with substitutable inputs than complementary inputs contrasts with the elasticities of substitution calibrated by Herrendorf et al. (2021), García-Santana et al. (2021) and Sposi et al. (2021), who use a single price for goods and a single price for services which implies input complementarity. If we aggregate goods and services prices across consumption, investment, and intermediates, we also find that services are getting more expensive relative to goods as a whole, because the relative price of services to goods is rising in consumption and intermediates. The reason for our different findings is aggregation bias. Because investment is a small fraction of output—on average over the period 1947-2020, investment is 21% and 5% of the gross output of goods and services, respectively—it is easy for aggregate price trends to be dominated by price movements in consumption and intermediates, masking the behavior of investment prices.

To highlight this argument, we consider an alternative construction of prices for goods and services by use, computed as a weighted average of gross output prices for the 40 consistent NAICS sectors in our industry data, with weights given by each sector's production share of consumption, investment, or intermediates within goods or services sectors (see Appendix C for details). With this approach, we find similar results for most of our price series, but we see sizable differences in the price of services investment. Although the price of services investment grows substantially less than the overall price of services when measured using gross output data, it still grows faster than goods investment, implying a rising relative price of services.

However, measuring the price of services investment using publicly available gross output prices is still subject to significant aggregation bias. The gross output price data can properly identify the price of services investment only if at least one of two conditions is met: 1) observed services sectors specialize in producing primarily one use (consumption, investment, or intermediates), implying no within-sector aggregation bias or 2) the price trends for consumption, investment, and intermediates within each sector are comparable. Neither of these conditions is satisfied at the 40-sector level of disaggregation. Even for the two services sectors that produce the

Figure 4: Heterogeneous Prices Within the Professional and Technical Services Sector



Notes: The thick line denotes the log gross output price for the professional and technical services sector, obtained from the BEA GDP by Industry database. The thin lines are the log prices of varied commodities produced by the professional and technical services sector. These detailed prices series are based on NIPA Tables 2.4.4U and 5.6.4.

most services investment (information and professional/technical services), only a small portion of gross output is used as investment (on average only 15% and 22%, respectively), suggesting the potential for significant aggregation bias.[14] Although the professional/technical services sector is the largest and fastest growing producer of services investment (primarily software and R&D), it also produces the consumption commodities of legal services and veterinary services (along with many other commodities).[15]

Using underlying detail in the national accounts (only available since 1959), Figure 4 plots the time series of the prices of these commodities compared to the gross output price of the professional/technical services sector as a whole. While there has been little price growth in the investment commodities produced by this sector (software and R&D), their price trends are masked by the rising prices of consumption commodities, such as legal services.[16]

In contrast, using expenditure-side data to measure prices makes aggregation bias less of a

[14]In more recent years, the GDP by Industry database has published price data for a more disaggregated set of industries. However, substantial evidence of potential aggregation bias persists. For example, in these data, the largest services-investment producer is miscellaneous professional/technical services, yet only 24% of its gross output is used for investment. See Appendix C for more details.

[15]Based on more detailed Input-Output data available only in 2007 and 2012, legal services and veterinary services are the two biggest consumption commodities and software and R&D are the two biggest investment commodities produced by this sector.

[16]In Appendix C, we provide similar evidence for the information sector, the other primary producer of services-investment.

concern. Using expenditure-side data, differences in consumption and investment prices between goods and services sectors are identified from heterogeneity in how much each sector produces different commodities (differences in ξ^C_{glt}, for consumption). These prices would be subject to aggregation bias if commodities are produced by an even mix of goods and services sectors; in the extreme case where each commodity is produced 50% by goods and services sectors each, prices of goods and services would be identical. However, nearly two-thirds of our commodities are produced in large majority (more than 80%) by either goods or services, implying that the bridge/make files at the level of commodity detail in expenditure side data sharply distinguish between goods and services production of consumption and investment.[17]

In summary, both expenditure-side and income-side price data can be used to measure goods and services prices by use. The best approach is the one that most precisely identifies price variation along the two dimensions that define them: whether the sector's output is a good or a service and whether the sector's output is used for consumption, investment, or as an intermediate input. Our evidence implies that the bridge/make files more precisely distinguish goods and services sectors on the expenditure side than the underlying sectoral detail on the income side distinguishes final use. Thus, expenditure-side data provide a more accurate measure of prices for goods and services sectors by use.

4. Model

To understand the observed patterns in sectoral expenditures and prices across production networks, we develop an N sector extension of the neoclassical growth model that explicitly incorporates the production networks for intermediate inputs and investment.

4.1. Technology

For each sector j, gross output, Q_{jt}, is produced using capital, K_{jt}, labor L_{jt}, and a bundle of intermediate goods M_{jt} according to the following Cobb-Douglas production function:

$$Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}, \tag{4}$$

[17]The main instances when both goods and services contribute significantly to the production of a commodity is when delivery of the commodity to the final user involves significant "margins" due to transportation, wholesale trade, or retail trade. If we reclassify these margin sectors as goods because they typically arise when the product is a physical good, then over 90% of all commodities would be produced in large majority by either goods or services. The patterns of relative prices we measure are robust to this reclassification (see Appendix C).

where A_{jt} is exogenous TFP in sector j.

Each sector's intermediates bundle, M_{jt}, is produced by an "intermediates bundling" sector for sector j, which aggregates intermediate goods from all sectors using a CES technology:

$$M_{jt} = A^M_{jt} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}, \tag{5}$$

where ϵ_{Mj} is the elasticity of substitution between intermediate inputs for sector j, $\omega_{Mij} \in [0, 1]$ (with $\sum_i \omega_{Mij} = 1$) determines the relative importance of inputs from each sector in producing intermediates, M_{ijt} represents intermediate inputs used in sector j purchased from sector i at time t, and A^M_{jt} represents exogenous intermediates-bundling TFP for sector j. We include this bundling TFP to allow for added flexibility when calibrating the model, but in Section 6 we show that movements in this term only have a modest effect on overall growth.

Capital is sector-specific and follows a standard law of motion, depreciating at rate δ_j:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + X_{jt}. \tag{6}$$

Investment, X_{jt}, is produced in an "investment bundling" sector for sector j's capital with the following aggregation technology:

$$X_{jt} = A^X_{jt} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}, \tag{7}$$

where ϵ_{Xj} is the elasticity of substitution between investment inputs in sector j, $\omega_{Xij} \in [0, 1]$ (with $\sum_i \omega_{Xij} = 1$) determines the relative importance of different inputs in producing investment, and X_{ijt} is investment inputs used in sector j purchased from sector i at time t. A^X_{jt} represents exogenous investment-bundling TFP for sector j, which is included for calibration flexibility.

4.2. Preferences

There is an infinitely lived representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t U(\{C_{it}\}_{i=1}^N), \tag{8}$$

where $0 < \beta < 1$ is the discount factor and C_{it} is consumption produced by sector i. We assume that the period utility function, $U(\{C_{it}\}_{i=1}^N)$ follows a log CES structure, with

$$U(\{C_{it}\}_{i=1}^N) = \ln \left(\left[\sum_i \omega_{Ci}^{1/\epsilon_C} C_{it}^{\frac{\epsilon_C-1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C-1}} \right), \tag{9}$$

where ϵ_C is the elasticity of substitution between consumption products and $\omega_{Ci} \in [0, 1]$ (with $\sum_i \omega_{Ci} = 1$) determines the relative importance of consumption products from each sector in aggregate consumption. Our framework abstracts from preferences over leisure, assuming the household inelastically supplies one unit of labor each period.

Our assumption of log CES preferences rules out income effects as a possible cause for structural transformation in consumption. However, given our focus on structural change in production networks, we make this assumption for ease of exposition and tractability; when we conduct our growth accounting exercises, our findings are robust to introducing income effects by adding non-homotheticities to household preferences (see Appendix E).

4.3. Equilibrium

We study the competitive equilibrium of this economy with representative profit-maximizing firms in all markets. The price of final output in each production sector j is denoted by P_{jt}; the price of the intermediates bundle is given by P^M_{jt}. The household owns the capital stock and accumulates capital in sector j by purchasing new investment goods from the investment bundling firm for sector j at price P^X_{jt}. The household rents sector-specific capital to each sector j at a rental price R_{jt}. Since labor is common to each sector and freely mobile, there is a single wage paid to the household, denoted by W_t. For reference, we provide a full listing of equilibrium conditions in Appendix D.

In equilibrium, all markets (final output, labor, capital, as well as intermediate and investment bundling) clear. Final output from each sector can be used as consumption for the household or as an input to intermediate and investment bundles across sectors, implying a market clearing relationship of:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{10}$$

The extent to which each sector produces consumption, intermediates, and investment depends on preference and bundling parameters; in Section 5, we consider a calibration in which each sector j specializes in the production of only one final use: consumption, investment, or intermediates.

With constant returns and competitive markets, the price indices for the bundle of intermediate goods, P^M_{jt}, and the bundle of investment goods, P^X_{jt}, are given by:

$$P^M_{jt} = \frac{1}{A^M_{jt}} \left(\sum_i \omega_{Mij} P_{it}^{1-\epsilon_{Mj}}\right)^{\frac{1}{1-\epsilon_{Mj}}}, \tag{11}$$

$$P^X_{jt} = \frac{1}{A^X_{jt}} \left(\sum_i \omega_{Xij} P_{it}^{1-\epsilon_{Xj}}\right)^{\frac{1}{1-\epsilon_{Xj}}}. \tag{12}$$

Furthermore, manipulating the first order conditions for each sector's production generates the following expression for the price of final output produced by each sector j:

$$P_{jt} = \frac{1}{A_{jt}} \left(\frac{R_{jt}}{\theta_j \alpha_j}\right)^{\theta_j \alpha_j} \left(\frac{W_t}{(1-\theta_j)\alpha_j}\right)^{(1-\theta_j)\alpha_j} \left(\frac{P^M_{jt}}{1-\alpha_j}\right)^{1-\alpha_j}. \tag{13}$$

Finally, we describe the equilibrium conditions determining structural change in production networks. From the first order conditions for the intermediates and investment bundling sectors, we derive the model expression for the shares of intermediates or investment spending by sector j on products made by sector i at time t, denoted s^M_{ijt} and s^X_{ijt}:

$$s^M_{ijt} \equiv \frac{P_{it} M_{ijt}}{P^M_{jt} M_{jt}} = \omega_{Mij} \left(\frac{P_{it}}{A^M_{jt} P^M_{jt}}\right)^{1-\epsilon_{Mj}}, \tag{14}$$

$$s^X_{ijt} \equiv \frac{P_{it} X_{ijt}}{P^X_{jt} X_{jt}} = \omega_{Xij} \left(\frac{P_{it}}{A^X_{jt} P^X_{jt}}\right)^{1-\epsilon_{Xj}}. \tag{15}$$

These intermediates and investment expenditure shares depend on the relative prices of each sector's output, and the CES scale (ω) and elasticity parameters (ϵ) in the bundling sectors. Movements in relative prices across sectors can thus induce structural change in production networks. The aggregate share of intermediates or investment spending on products made by sector i, s^M_{it} and s^X_{it} is given by a weighted average of these sectoral expenditure shares, following the same relation described for the empirical work in equation (1). As in Section 2, these expressions show that changes in production networks in the model can reflect both changes in production processes within sectors (changes in s^M_{ijt} and s^X_{ijt}) and changes in the composition of spending across sectors.

4.4. Balanced Growth Path

To develop intuition for the connection between structural change and economic growth, we consider an aggregate balanced growth path (ABGP) of the model. We use the same definition of an ABGP as Ngai and Pissarides (2007) and Herrendorf et al. (2021), where all aggregates denoted in units of the numeraire (aggregate investment) must grow at a constant rate. The existence of such a growth path requires several assumptions, but we do not impose these assumptions for the calibration and quantitative exercises in Sections 5 and 6. By measuring aggregates in units of aggregate investment, this means that TFP growth in the production of consumption will not impact aggregate GDP (as in Ngai and Pissarides, 2007; Herrendorf et al., 2021). However, our quantitative exercises measure GDP in a more data-consistent fashion (as a chained index number); the intuition we develop here regarding investment and intermediates structural change and economic growth also applies to consumption structural change in those exercises.

To characterize the ABGP, we impose the following assumptions:[18]

Assumption 1. *The parameters of sectoral production functions are the same across all sectors, i.e.,* $\alpha_j = \alpha$ *and* $\theta_j = \theta$ *for all* j*.*

Assumption 2. *The parameters governing the evolution of capital—both parameters of the investment bundling sectors and the depreciation rate—are the same for all sectors* j*, i.e.,* $\delta_j = \delta$*,* $\omega_{Xij} = \omega_{Xi}$ *and* $\epsilon_{Xj} = \epsilon_X$ *for all* j*. Furthermore, each sector's investment bundling TFP is the same, i.e.,* $A^X_{jt} = A^X_t$*.*

Assumption 3. *The parameters for intermediates bundling are the same for all sectors and each sector's intermediates bundling TFP is the same, i.e.,* $\omega_{Mij} = \omega_{Mi}$*,* $\epsilon_{Mj} = \epsilon_M$*, and* $A^M_{jt} = A^M_t \ \forall j$*.*

For an ABGP to exist, we must impose assumptions that ensure homogeneity in production functions and investment bundling across sectors, thereby allowing us to define an aggregate production function.[19] While assumption 3 is not necessary for the existence of an ABGP, we impose it to allow for closed-form solutions that facilitate a more transparent discussion of the model's implications. For any variable Z_t, we define the gross growth rate between time periods t and $t+1$ as $\gamma^Z_{t+1} \equiv \frac{Z_{t+1}}{Z_t}$. Along the ABGP, we omit time subscripts for variables growing at a constant rate. Using these definitions, assumptions 1 – 3 imply the following lemma and proposition:

[18] An important assumption already embodied in the model is that of log preferences over consumption (i.e., a unitary intertemporal elasticity of substitution). For a deeper description of the impact of this assumption, see important recent work by Buera, Kaboski, Mestieri and O'Connor (2020).

[19] See Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado, Van Long and Poschke (2017) for examples of models with differential capital and labor intensities that allow for balanced growth in limiting cases.

Lemma 1. *Under assumptions 1 – 3, aggregate GDP, denoted in units of the numeraire (aggregate investment), is given by:*

$$Y_t = \sum_i P_{it}^V V_{it} = \sum_i \left(P_{it}Q_{it} - P_{it}^M M_{it}\right) = \mathcal{A}_t K_t^{\theta}, \tag{16}$$

where V_{it} represents real value added in sector i, P_{it}^V is the price of value added, $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}}$ where $B_t^X \equiv A_t^X \left(\sum_k \omega_{Xk} A_{kt}^{\epsilon_X - 1}\right)^{\frac{1}{\epsilon_X - 1}}$ and $B_t^M \equiv A_t^M \left(\sum_k \omega_{Mk} A_{kt}^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}}$, and $K_t = \sum_i K_{it}$.

Proof. See Appendix D. □

Proposition 1. *Assume that assumptions 1 – 3 hold and $\left(\gamma_t^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} > \beta(1-\delta)\,\forall t$. Then, with aggregate investment as the numeraire, an aggregate balanced growth path exists if and only if $\gamma_t^{\mathcal{A}}$ is constant. Furthermore, along that path:*

$$\gamma^K = \gamma^X = \gamma^Y = \gamma^W = \gamma^{E^C} = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}, \tag{17}$$

and $\gamma^R = 0$, where E_t^C represents consumption expenditures in terms of the numeraire, with $E_t^C \equiv \sum_i P_{it} C_{it}$

Proof. See Appendix D. □

Aggregate GDP has the same Cobb-Douglas structure as sectoral real value added (with labor dropping out because it is in unit aggregate supply). With investment as the numeraire, aggregate TFP, $\mathcal{A}_t$, only depends on TFP growth that expands the frontier of investment production, either through TFP growth stemming directly from investment producers (B_t^X) or TFP growth originating from their intermediate suppliers (B_t^M).[20] Given the aggregate production function (16), the equalization of growth rates in equation (17) is unsurprising as the economy's aggregate growth rate only depends on the growth rate of aggregate TFP. The requirement that $\left(\gamma_t^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} > \beta(1-\delta)$ is standard and holds for most reasonable parameter values.

The ABGP in our model is similar to that of Ngai and Pissarides (2007) in that the aggregate growth rate only depends on technological change that increases the production frontier for investment. However, our aggregate balanced growth path differs in that it allows for structural change

[20]Without assumption 3, the model still admits an AGBP but it is not possible to separate these two components of aggregate TFP. Heterogeneity in how different investment producers source their intermediates implies that aggregate growth depends on the specific input-output linkages for each investment producer. Our quantitative exercises, allow for such heterogeneity.

in production networks. The key distinction between our frameworks lies in the assumptions made about TFP growth. With constant productivity growth in all sectoral TFP terms, A_{it}, assumed by Ngai and Pissarides (2007), structural change in production networks is inconsistent with balanced growth. In contrast, our balanced growth path is generated from the assumption that the combination of TFP growth in all sectors (including bundling sectors) generates constant growth in aggregate TFP, $\mathcal{A}_t$ (see Herrendorf et al. (2021) for further discussion).

While our ABGP is a special case that requires strong assumptions, our primary interest lies in tightly characterizing the composition of aggregate TFP growth, $\mathcal{A}_t$, and how this composition varies along the ABGP.[21] By definition, structural change cannot impact the aggregate growth rate along the ABGP, but a decomposition of aggregate TFP growth along the ABGP builds intuition for the channels through which structural change can affect growth.

The (scaled) growth rate of aggregate TFP, and thus the growth rate of aggregate GDP along the ABGP, is given by $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} = \left(\gamma_t^{B^X}\left(\gamma_t^{B^M}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\theta}}$. Because the aggregate growth rate depends on both $\gamma_t^{B^X}$ (the direct effect of TFP growth originating from investment producers) and $\gamma_t^{B^M}$ (the indirect effect of TFP growth stemming from intermediates suppliers), there is a direct connection between the aggregate growth rate and structural change. To see this, consider the model expressions for these two sources of aggregate growth along the ABGP:

$$\gamma_t^{B^M} = \gamma_t^{A^M}\left(\sum_i s_{it-1}^M(\gamma_{it}^A)^{\epsilon_M-1}\right)^{\frac{1}{\epsilon_M-1}}, \tag{18}$$

$$\gamma_t^{B^X} = \gamma_t^{A^X}\left(\sum_i s_{it-1}^X(\gamma_{it}^A)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}. \tag{19}$$

In addition to the Cobb-Douglas parameters on capital (θ) and value-added (α), equations (18) and (19) illustrate that the growth rate of aggregates depends on four components: growth in intermediates-bundling TFP ($\gamma_t^{A^M}$), growth in investment-bundling TFP ($\gamma_t^{A^X}$), and two weighted sums of TFP growth in each individual production sector i, where the weights are determined by the composition of intermediates and investment production (s_{it}^M and s_{it}^X).

Differential TFP growth across sectors, which generates differential time paths of relative prices, can induce structural change in production networks—captured by changes in s_{it}^M and s_{it}^X. Equations (18) and (19) highlight that the impact of this structural change on the composition of ag-

[21] Alternatively, Buera et al. (2020) develop methods to study the "stable transformation path" in models that depart from balanced growth, which could be used to examine these relationships more generally.

gregate growth critically depends on the elasticities of substitution in the CES aggregators: ϵ_M for intermediates and ϵ_X for investment. Lemma 2 formalizes this relationship:

Lemma 2. *Assume that assumptions 1-3 hold. Further, assume that there is weak positive dependence between the log of sectoral TFP and TFP growth across sectors (formally,* $\mathbb{E}\left[ln(A_{it-1}) \mid \gamma_{it}^{A} = a\right]$ *is weakly increasing in a with a probability measure across sectors of s_{it-1}^{M}).*

Holding all other parameters fixed, the growth rate of γ_t^{BM} is then weakly increasing in ϵ_M for all time periods t. The same result holds for the growth rate γ_t^{BX}; all other parameters fixed, it is increasing in ϵ_X for all time periods t.

Proof. See Appendix D. □

Thus, the higher the elasticity of substitution is for the investment or intermediates network, the faster the composite of TFP growth in that network, γ_t^{BX} or γ_t^{BM}, will grow. The intuition for this result can be seen from the limiting cases when sectoral TFP growth rates are constant over time.[22] For example, if $\epsilon_M < 1$ (i.e., gross complements), then as $t \to \infty$, γ_t^{BM} will converge to the *slowest* TFP growth rate among producers of intermediates. This is because movements in relative prices will ultimately cause s_{it}^{M} to converge to unity for the slowest growing sector. In contrast, in the case of gross substitutes (i.e., $\epsilon_M > 1$), this growth rate will converge to the *fastest* TFP growth rate, as all expenditures ultimately become concentrated on this sector.

Furthermore, when one production network features complementarity in sectoral inputs and the other features substitutability, the network that features gross substitutability will become endogenously more important for aggregate growth over time. Resources are reallocated toward the fastest-growing producers in the network with substitutability ("frontiers"), while resources are allocated to the slowest-growing producer in the network with complementarity ("bottlenecks"). Thus, structural change in production networks can affect the composition of aggregate growth along the balanced growth path and potentially the level of aggregate growth when balanced growth

[22]Lemma 2 does not require that sectoral TFP growth rates be constant over time. However, the weak positive dependence assumption is trivially satisfied if sectoral TFP growth rates are constant over time (especially if all TFP series are initially normalized to 1, though that is not required). The lemma requires that there is weak positive dependence, which imposes that, on average, sectors with high TFP today are also growing fast tomorrow. This imposes some stability in relative growth rates across sectors over time.

assumptions are relaxed. The remainder of the paper focuses on the model's implications for aggregate growth when we study GDP as an index number and do not impose these assumptions.

5. Calibration, Transition Path, and Structural Change

To examine the quantitative implications of our model, we calibrate all model parameters and numerically solve the model's transition path. A key challenge for calibration is determining the appropriate level of aggregation. On the one hand, we document in Section 2 that the primary patterns of structural change in production networks can be summarized using just two sectors: goods and services. On the other hand, in Section 3 we emphasize significant heterogeneity in relative price trends across consumption, investment, and intermediate input producers within these two broad sectors.

Thus, to accommodate this heterogeneity in price trends, we focus on a six-sector aggregation, with each sector defined by the interaction of its product market—goods or services—and the eventual use of these products—as final consumption, investment, or an intermediate input. The resulting six sectors are goods-consumption (e.g., books, toys, food), goods-investment (e.g., buildings, machines, vehicles), goods-intermediates (e.g., primary metals, chemicals), services-consumption (e.g., education, health care), services-investment (e.g., software, R&D), and services-intermediates (e.g., financial services, wholesale trade). Consequently, our calibration assumes that each sector in the model fully specializes in the production of either consumption, investment, or intermediates.

5.1. Calibration: Non-CES Parameters

We calibrate the non-CES aggregator parameters using BEA data from 1947-2020 at the 40 sector level. For each parameter $(\alpha_j, \theta_j, \delta_j)$, we first obtain implicit values at the 40-sector level and then aggregate those values to the six sectors in our calibration. This procedure is potentially subject to the aggregation bias described in Section 3, but the expenditure-side national accounting approach we developed for price measurement is not feasible for these parameters. Furthermore, we are less concerned about the impact of aggregation bias for these parameters as they are not allowed to change over time in our model and because our quantitative results are robust to setting all production parameters to common values across sectors.

We calibrate α_j using each sector's ratio of nominal value added to nominal gross output, θ_j

Table 1: Six-Sector Calibrated Parameters

	θ	α	δ	ω_{Xg}	ϵ_X	ω_{Mg}	ϵ_M
Goods Sectors							
Consumption	0.36	0.36	0.09	0.81	4.10	0.81	0.29
Investment	0.21	0.46	0.12	0.79	3.40	0.71	0.52
Intermediates	0.35	0.42	0.09	0.82	3.81	0.77	0.45
Services Sectors							
Consumption	0.37	0.69	0.05	0.80	1.80	0.44	0.00
Investment	0.40	0.68	0.09	0.72	2.84	0.34	0.00
Intermediates	0.37	0.66	0.07	0.81	2.49	0.32	0.00

Notes: This table reports the calibrated parameter values for the Cobb-Douglas exponents in production (θ for capital within value-added, α for the value-added portion of gross output) and the depreciation rate of capital in each of the six sectors we study. The table also reports the calibrated parameter values for the CES aggregators corresponding to equations (5), (7), and (9). For each use (consumption, investment, intermediates), the parameters ϵ_X and ϵ_M represent the elasticity of substitution between goods and services inputs within investment and intermediates, and the parameters ω_{Mg} and ω_{Xg} represent the share parameter attached to goods inputs (with $1 - \omega$ being the share parameter for services) within investment and intermediates.

using one minus the ratio of nominal labor compensation to nominal value added (minus taxes and adjusted for self-employment), and δ_j using the implied depreciation rates for each NAICS sector published in the BEA Fixed Assets database. We aggregate to six sectors using a weighted average of the 40-sector parameter values, where each sector is weighted by its production of each use—consumption, investment, or intermediates—as a fraction of total goods or services production.[23] We average these sectoral parameter values over the entire sample 1947-2020 and use the average as the calibrated value of each parameter.

Table 1 reports the resulting calibrated parameters. The most notable heterogeneity in these parameters across sectors is that α_j is lower for goods sectors, as goods sectors are much more intermediates intensive than services sectors. Also, θ_j is relatively low for the goods-investment sector, reflecting the high labor share of the construction sector that contributes to goods-investment.

5.2. *Calibration Strategy: CES Parameters*

To calibrate the CES aggregator parameters for investment and intermediates—the share parameters (ω_{Xij}, ω_{Mij}) and the elasticity parameters (ϵ_j^X, ϵ_j^M)—we first construct the share of goods and services investment and interemdiates for each of our six sectors. Similar to the calibration targets for non-CES parameters, we generate these by aggregating intermediate and investment expenditure patterns across the 40 BEA sectors in proportion to each sector's role in producing consumption, investment, or intermediates within goods or services. Again, there is a potential

[23] For example, to get parameter values for the goods-consumption sector, we weight parameter values for each goods subsector k with the weight $\frac{P_{kt}C_{kt}}{\sum_{l \in \text{Goods}} P_{lt}C_{lt}}$.

that this approach is subject to aggregation bias. As we do not have sufficient data to rule out this possibility, it is a question for further research. However, our results are robust to assuming all sectors have the same CES technologies with common elasticities of substitution for investment and intermediates.

Using these BEA shares, we calibrate the CES aggregator parameters (including those for consumption) in two steps. First, for each sector, j, the share parameters are calibrated so that the model matches the initial fraction of expenditures on consumption, intermediates, or investment purchased from sector i in the year 1947. Then, we combine equations (14) and (15) with equations (11) and (12) (and analogous equations for consumption) to obtain the following equations used to calibrate the elasticities of substitution:[24]

$$s_{gjt}^{M} = \frac{\omega_{Mgj} P_{g-m,t}^{1-\epsilon_{Mj}}}{\omega_{Mgj} P_{g-m,t}^{1-\epsilon_{Mj}} + (1-\omega_{Mgj}) P_{s-m,t}^{1-\epsilon_{Mj}}} \tag{20}$$

$$s_{gjt}^{X} = \frac{\omega_{Xgj} P_{g-m,t}^{1-\epsilon_{Xj}}}{\omega_{Xgj} P_{g-x,t}^{1-\epsilon_{Xj}} + (1-\omega_{Xgj}) P_{s-x,t}^{1-\epsilon_{Xj}}} \tag{21}$$

$$s_{gt}^{C} = \frac{\omega_{Cg} P_{g-c,t}^{1-\epsilon_{C}}}{\omega_{Cg} P_{g-c,t}^{1-\epsilon_{C}} + (1-\omega_{Cg}) P_{s-c,t}^{1-\epsilon_{C}}} \tag{22}$$

where g–c, g–x, g–m, s–c, s–x, s–m identify our six sectors, g and s are goods and services, and c, x, m are consumption, investment, and intermediates. Given calibrated values for all ω parameters, we calibrate the elasticity parameters using equations (20), (21), and (22), minimizing the least squares differences between the model series for structural change and the data series for structural change in each of our six sectors over the years 1947-2020.

Parameter values for the investment and intermediates bundling technologies are reported in Table 1. Consistent with our price measurement, the calibrated values for the elasticity parameters confirm that goods and services are complements in the production of intermediates in all sectors (elasticities less than one), but substitutes in investment production in all sectors (elasticities greater than one). For consumption, our calibration yields a goods share (ω_{Cg}) of 0.31 and an elasticity of substitution (ϵ_C) of 0.00. The elasticities for consumption and intermediates imply strong complementarity between goods and services, consistent with existing literature (e.g., Herrendorf et al., 2021; García-Santana et al., 2021; Sposi et al., 2021). However, the best fit for patterns

[24]That is, to obtain these calibration equations, we need the model expressions for CES bundling technologies in consumption, investment, and intermediates, optimization by the household and firms, and perfect competition. The remaining parts of the model are not needed to obtain these equations we use to calibrate elasticities of substitution.

of structural change in intermediates within the three goods sectors implies less complementarity than the Leontief specification.

The six-sector calibration fit for structural change in intermediates and investment can be found in Appendix E; the model's overall fit to structural change depends on the aggregation of structural change in each sector, so we assess the fit after we calibrate and solve the transition path.

5.3. Transition Path and Calibrated TFP Series

With heterogeneity in production technologies and investment bundling across sectors, the assumptions necessary for balanced growth do not hold. As a result, we solve for the model's transition path between two steady states. Furthermore, with parameter heterogeneity, relative sectoral prices also depend on wages and sectoral rental rates of capital. Thus, for the model to match the relative prices used in calibrating the CES bundling technologies requires that some TFP series be calibrated along the transition path. First, we describe how we calibrate the TFP series, then we describe the assumptions used in solving the transition path.

The model has two types of TFP: TFP in each of the six sectors' production technologies (sectoral TFP) and TFP in the bundling technologies (bundling TFP) for intermediate and investment inputs. For five of our six sectoral TFP series, we calibrate TFP by matching relative output prices in the model to relative output prices in the data. We calibrate TFP for the final sector to match the aggregate series for GDP per worker, given by a Tornqvist aggregate of sectoral real value added (from the 40 BEA sectors) divided by total employment (measured as the sum of sectoral employment as recorded in the BEA's Industry Accounts and NIPA Table 6.4).[25]

We measure growth in intermediates and investment bundling TFP using a log first-order approximation of the equilibrium intermediate and investment input price indices (equations (11) and (12)), for years t and $t-1$. When log-linearized around the average expenditure share in these two years, the resulting Tornqvist index can be rearranged to solve for the growth in bundling TFP. For example, intermediates bundling TFP for sector i is computed as:

$$\Delta \ln(A_{it}^{M}) = -\left(\Delta \ln(P_{it}^{M}) - \sum_{j=g-m,s-m} \left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^{M}M_{it}}}\right) \Delta \ln(P_{jt})\right), \tag{23}$$

where g–m and s–m are the goods-intermediates and services-intermediates sectors and $\left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^{M}M_{it}}}\right)$

[25] If we instead calibrate the final TFP series to match aggregate TFP growth, we obtain very similar results.

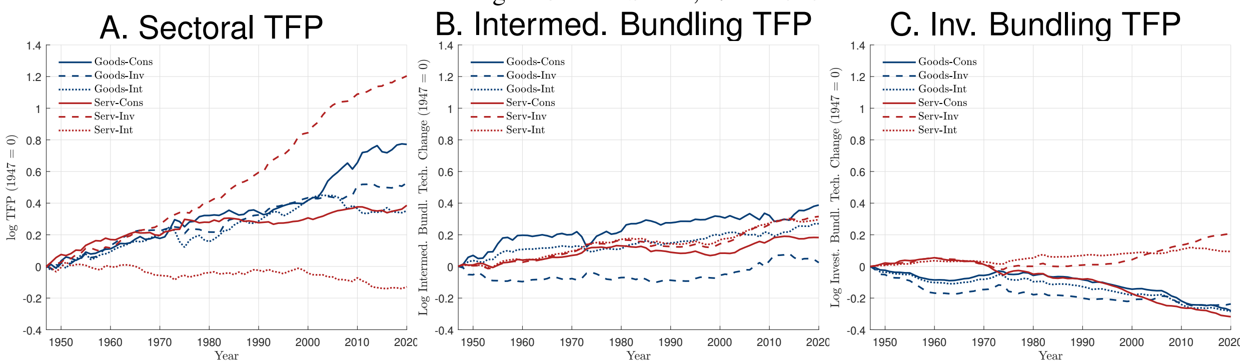
is the average between t and $t-1$ of the share of intermediate spending by sector i purchased from sector j. We measure $\Delta \ln(P_{it}^{M})$ using BEA GDP by Industry data on the price of intermediate bundles by sector, aggregated to the six-sector level. We use a similar equation to identify investment bundling TFP, with the sectoral investment price index constructed as a Tornqvist index of the price of different investment commodities in the NIPA, weighted by sectoral investment spending on each commodity (as reported in the BEA Fixed Assets database and used in the construction of the invesmtent network).

Using this approach to measure the bundling TFP series simplifies our calibration as it is done independently of the model's calibrated parameters and fit. The relative values of intermediates and investment bundle prices along the transition path closely match those observed in the data, suggesting that the cost of this approximation is negligible. This is perhaps unsurprising as we expect the growth in these bundling TFP series to be small, as they are a residual of observed intermediates bundle prices. That is, they largely consist of aggregation error leftover in intermediate input and investment bundle prices after accounting for the average price of investment inputs produced by goods and services sectors.

To solve the transition path of the model, we assume that the model begins in a steady state determined by the initial conditions in all exogenous TFP processes. We then solve the transition path under the following assumptions regarding the evolution of the TFP series. We first assume that TFP values remain constant for 20 years (a discarded burn-in period to account for the news about future TFP paths) and then evolve according to the data from 1947 to 2020. We assume that for the subsequent 50 years, 2020-2070, all TFP series grow at a constant rate given by each series' average growth rate for the prior 10 years. After 2070, growth in bundling TFP series and calibration targets gradually slows to zero over the next 20 years and remains constant for another 40 years, implying that the model attains a terminal steady state in the year 2130. However, results are not sensitive to varying the lengths of any of the burn-in, slowdown, or constant periods.

Figure 5 displays calibrated sectoral TFP (panel A) and measured intermediates and investment bundling TFP (panels B and C) for each sector (in logs). Given that sectoral TFP is calibrated to match relative price data, it is unsurprising that growth in sectoral TFP follows nearly the opposite ranking of growth in each sector's observed prices in panel A of Figure 3. That said, there are

Figure 5: TFP Series, 1947-2020



Notes: Panel A shows log sectoral TFP over time for each sector, A_{it}; panel B shows log intermediates-bundling TFP over time, A^M_{it}; panel C shows log investment-bundling TFP over time, A^X_{it}. In the figures, all log TFP series are normalized to 0 in 1947.

some notable differences between these series and observed price dynamics. For example, because services sectors use much larger shares of services intermediates whose prices have been rising rapidly, the underlying technology growth in services sectors is faster than what is observed with relative prices alone. This explains why TFP in services investment is significantly higher than in goods consumption, despite the two sectors showing very similar price patterns. This highlights the importance of accounting for underlying production networks in accurately identifying sectoral TFP growth.

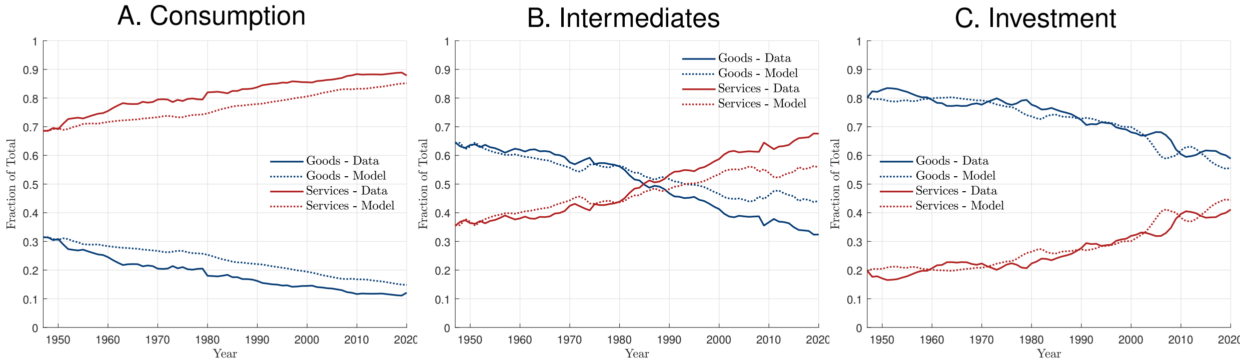
On average, intermediates and investment bundling TFP growth is low (0.3% and -0.2% a year, respectively, averaged across sectors). As we show in our growth accounting exercises below, these series play only a minor role in generating economic growth in the postwar period.

5.4. Structural Change

We next solve for aggregate structural change along the transition path and present the model's fit to patterns of structural change in consumption, investment, and intermediates in Figure 6.[26] Overall, the calibrated model provides a good fit to patterns of structural change, though a few comments are warranted. First, although the model accounts for much of the structural change in

[26]In our model, all sectoral production technologies are Cobb-Douglas, implying a constant share of spending on intermediate inputs over time. For consistency, the data series for aggregate structural change in each of our six sectors in Figure 6 is constructed with the assumption of a constant share of intermediates spending within sectors. Specifically, aggregation weights are given by the product of gross output and the average ratio of intermediates spending to gross output. This generates slightly different empirical patterns of structural change than seen in Figure 2. The model fit for each of the six sectors' intermediates expenditure patterns is shown in Appendix E.

Figure 6: Model Calibration Fit to Structural Change Patterns in Consumption, Intermediates and Investment, 1947-2020



Notes: The figures plot the fraction of total spending on consumption, intermediates, and investment produced by the goods sector (blue lines) and the services sector (red lines). Data series are solid lines; model series are dashed lines.

consumption, the model does not match the entire rise in the share of services. This is unsurprising, given that household preferences in our model do not feature any income effects, which are important for explaining structural change in consumption (e.g., Boppart, 2014; Comin, Lashkari and Mestieri, 2021). In Appendix E, we extend our model to allow for non-homotheticities in preferences. While this extension allows us to match structural change in consumption exactly, it has minimal impact on our growth accounting results. Second, the model closely matches the long-run patterns in structural change in investment. Given that we abstract from adjustment costs and uncertainty about the future price of investment, it is not surprising that the model does not generate some of the short and medium-run fluctuations observed in the data.[27]

Finally, the model reproduces about 2/3 of the rising share of intermediates produced by services. The model fit is even stronger before 2008, explaining roughly 80% of the overall increase, but fails to capture a substantial portion of the increased share of services intermediates thereafter. However, despite the imperfect aggregate fit to intermediates structural change, we show in Appendix E that the model results reproduce the contribution of within-sector and between-sector

[27] As noted in vom Lehn and Winberry (2022), introducing an investment network into a multi-sector growth model tends to generate a counterfactually volatile distribution of investment expenditures. In our framework, this volatility interacts with heterogeneity in each sector's share of investment purchased from goods to generate volatility in the aggregate structural change in investment. Since we are interested in the trend behavior of structural change, we present aggregate structural change in investment with a smoothed share of investment expenditures by each sector to mitigate this volatility. Alternatively, we obtain similar results if we smooth all TFP series and calibration targets. Given our interest in long run outcomes, our subsequent results are not substantively affected by either this short-run volatility or the alternative of smoothing all driving forces fed into the model.

forces to structural change in intermediates. While the imperfect fit of the model may reflect the presence of income effects (i.e., scale effects), it may also reflect data limitations or additional heterogeneity in intermediates prices beyond what we observe in the data. Given the challenges in measuring intermediates prices, the model does a good job of capturing the overall pattern of structural change in intermediates.

6. Growth Accounting along the Transition Path

We now conduct two sets of growth accounting exercises along the transition path of our model. First, we decompose the historical growth rate of aggregate GDP for the U.S. into growth generated by consumption-specific, investment-specific, and intermediates-specific technical change. We also analyze counterfactual transition paths with no structural change in consumption, intermediates, or investment to assess the role of structural change in generating economic growth over time. Second, we consider projections of U.S. economic growth based on different scenarios of structural change. In particular, substitutability in investment inputs reduces projected future growth slowdowns related to Baumol's cost disease (Baumol (1967)).

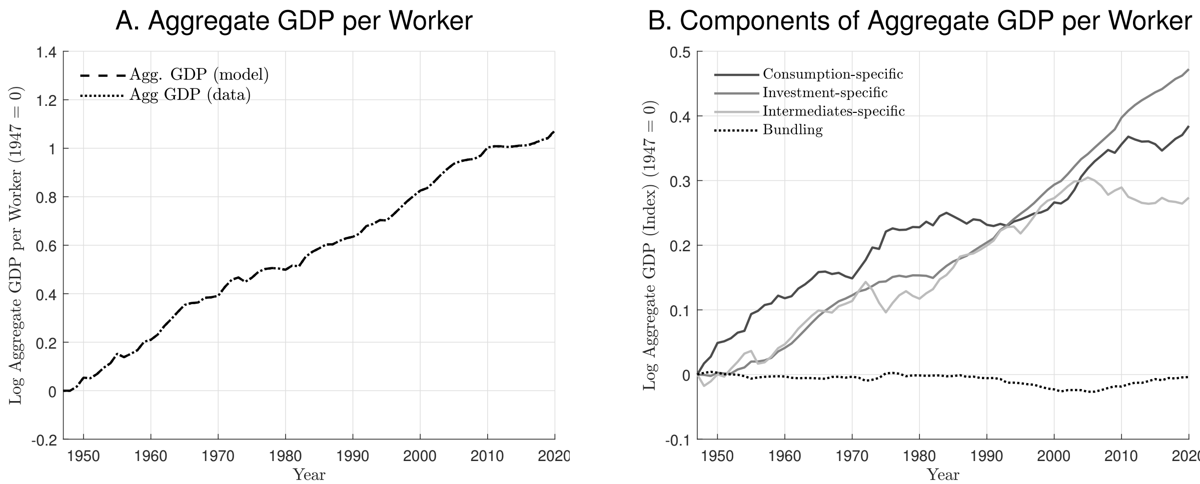
6.1. Decomposing Historical Growth

From the transition path, we compute the time series for aggregate GDP as a Tornqvist index, Y_t, similar to what is done in national accounts; this is identical to aggregate GDP per worker in our model since labor supply is fixed in unit supply.[28] The left panel of Figure 7 plots the log of aggregate GDP per worker in the model and the data (normalized to 0 in 1947); given our calibration strategy, these match exactly. GDP per worker grows at a fairly steady rate over most of the sample but shows a pronounced slowdown since 2010.

Panel B of Figure 7 shows four counterfactual series for the log of GDP per worker based on transition paths where only a subset of TFP series grow. First, we allow only TFP growth in goods-consumption and services-consumption ("consumption-specific technical change"), holding all other TFP series fixed at their initial values. Second, we allow only TFP growth in goods-investment and services-investment sectors, as well as in investment bundling TFP ("investment-

[28]The U.S. national accounting system uses a Fisher ideal index number instead of a Tornqvist index, however, our model is calibrated to match GDP per worker measured using the latter. The two indices produce nearly identical results for U.S. data.

Figure 7: Aggregate GDP per Worker Growth and its Composition, 1947-2020



Notes: Panel A shows the time series of aggregate GDP per worker in the model and in the data, both measured in logs (normalized to zero in 1947); panel B shows counterfactual evolutions of aggregate GDP per worker for four cases: (1) only TFP growth among consumption producers ("Consumption-specific"), (2) only TFP growth among investment producers and in investment-bundling ("Investment-specific"), (3) only TFP at intermediates producers and from intermediates bundling technical change ("Intermediates-specific"), and (4) only TFP growth in investment bundling and intermediates bundling ("Bundling").

specific technical change"). Third, we allow only TFP growth in the goods-intermediates and services-intermediates sectors, as well as in intermediates bundling ("intermediates-specific technical change"). Finally, we construct GDP per worker only using investment and intermediates bundling TFP. These bundling TFP series are already included in investment- and intermediates-specific technical change, but we consider them separately to illustrate their limited contribution to overall growth. Table 2 presents the corresponding numerical growth decomposition both for the entire sample and for three (approximately) twenty-year intervals beginning in 1960; decadal numbers beginning in 1950 are in Appendix E.

Based on this decomposition of GDP growth, we make three observations about the sources of aggregate GDP growth. First, all three sources of technical change – consumption, investment, and intermediates – significantly contribute to aggregate growth. From Table 2, over the entire sample of 1947-2019, consumption-specific technical change accounts for 36%, investment-specific technical change accounts for 44%, and intermediates-specific technical change accounts for 25% of aggregate GDP growth.

Second, for most of the sample, technical change in investment and intermediates bundling contributes very little to aggregate GDP growth; Table 2 shows that this source of technical change

Table 2: GDP Growth Decomposition, 1947-2019

	Aggregate GDP per Worker Growth: $\Delta\ln(Y_t)$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of GDP Growth	$\Delta\ln$	%	$\Delta\ln$	%	$\Delta\ln$	%	$\Delta\ln$	%
All	1.04	100	0.29	100	0.33	100	0.22	100
Investment-Specific	0.47	44	0.11	39	0.14	43	0.17	78
Intermediates-Specific	0.26	25	0.07	24	0.16	48	-0.01	-4
Consumption-Specific	0.37	36	0.11	38	0.04	12	0.10	48
Bundling	-0.00	0	0.00	1	-0.02	-7	0.02	9

Notes: The table shows long-run log changes in aggregate GDP per worker across different periods for five alternative simulations: (1) the full model simulation with all measured TFP series (calibrated to match the data exactly) and counterfactual simulations with TFP growth only from (2) investment producers and exogenous investment-bundling TFP ("investment-specific technical change"), (3) intermediates producers and exogenous intermediates-bundling TFP ("intermediates-specific technical change"), (4) consumption producers ("consumption-specific technical change"), and (5) both investment-bundling and intermediates-bundling TFP. Counterfactual changes are expressed as a percent of the change from the full model; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. Furthermore, the bundling TFP series are already included in the investment- and intermediates-specific technical change counterfactuals, so adding their percentage contribution will overstate total growth.

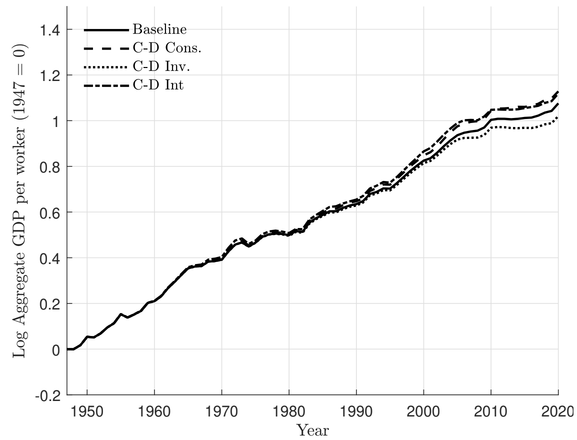
contributes almost nothing to total growth since 1947. However, it has recently become slightly more important, accounting for 9% of GDP growth since 2000. In Appendix E, we separately decompose the contribution of intermediates and investment bundling TFP, and find that intermediates bundling is slightly more important.

Third, the importance of investment-specific technical change is rising over time. Table 2 shows that this source of technical change accounts for roughly 40% of aggregate growth before 2000 and for more than 75% since 2000. Furthermore, Figure 7 and Table 2 highlight that intermediates-specific technical change stagnates and even declines after 2000.

As discussed in Section 4.4, the rising importance of investment-specific technical change could reflect either changing TFP growth rates or the endogenous reallocation of resources across sectors. To explore the importance of endogenous reallocation, we consider counterfactuals where we use all calibrated TFP series, but solve the transition path assuming one of consumption, investment, or intermediates bundling technologies are Cobb-Douglas. This assumption rules out structural change in that use. Figure 8 presents the counterfactual aggregate GDP series and Table 3 reports the contribution of reallocation forces to aggregate GDP growth (100% minus the percent contributions reported in the second column of each panel in the table).

As expected, when the aggregation of investment inputs is Cobb-Douglas, growth is lower,

Figure 8: Aggregate GDP Growth, Cobb-Douglas Counterfactuals, 1947-2020



Notes: This figure shows GDP growth for the baseline transition path of the model and three counterfactuals – where consumption, investment, or intermediates aggregation is assumed to be Cobb-Douglas in place of CES. In each simulation, the same TFP series are fed in, but the Cobb-Douglas assumption shuts down structural change in either consumption, investment, or intermediates.

Table 3: GDP Growth, Cobb-Douglas Counterfactuals, 1947-2019

	GDP Growth: $\Delta \ln Y_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of GDP Growth	$\Delta \ln$	%	$\Delta \ln$	%	$\Delta \ln$	%	$\Delta \ln$	%
Baseline	1.04	100	0.29	100	0.33	100	0.22	100
Consumption Cobb-Douglas	1.10	105	0.29	102	0.34	105	0.25	115
Investment Cobb-Douglas	0.99	95	0.29	99	0.32	98	0.17	80
Intermediates Cobb-Douglas	1.08	104	0.30	103	0.36	109	0.22	101

Notes: The table reports log changes in aggregate GDP per worker, $\ln(Y_t)$, across different periods under different assumptions regarding CES bundling technologies. The table reports counterfactuals for the cases where consumption, investment, or intermediates aggregation is Cobb-Douglas, ruling out structural change in that commodity. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

as resources no longer reallocate to the fastest growing producer of investment. Over the entire sample, Cobb-Douglas investment aggregation implies 5% lower total growth in GDP per worker, but this effect is much larger since 2000, where growth is 20% lower. When either intermediates or consumption is Cobb-Douglas, growth is faster, as resources no longer reallocate toward the slowest growing producer. Over the entire sample, the effect of complementarity in aggregation in either of these products is to reduce GDP growth by about 5%, though the effect attributable to consumption since 2000 is 15%.[29]

[29] In Appendix E, we consider how these results change when we allow for income effects in preferences, which reduces the amount of complementarity between goods and services in consumption. As a result, the effects of

These findings lead us to three observations about U.S. economic growth and its recent slowdown. First, investment-specific technical change is an increasingly important force in generating economic growth and structural change in investment has played a substantial role in driving economic growth since 2000. Second, structural change in consumption and intermediates have exerted a modest drag on aggregate growth, and the drag from consumption has been larger since 2000. Finally, intermediates-specific technical change has stagnated, with a negative total contribution to growth since 2000, and appears to be a significant contributor to the recent slowdown in economic growth. Thus, although significant attention has been paid to bottlenecks in supply chains since the COVID-19 pandemic, we find substantial sluggishness along this network in the years preceding the pandemic.

6.2. GDP Growth Forecasting and Baumol's Cost Disease

A long-standing concern with structural change, going back at least to Baumol (1967), is that if resources reallocate to the slowest growing producers in an economy, economic growth will stagnate. In our model, strong complementarity between goods and services as inputs for producing consumption and intermediates implies that structural change in these sectors reduces economic growth. However, our finding of substitutability for goods and services as inputs in investment production has the opposite effect: structural change in this sector boosts economic growth. To assess the relative strength of these forces, we analyze our model's predictions for future economic growth under a variety of scenarios.

To forecast future growth, we assume that all TFP series grow at a constant rate from 2021-2070 along the transition path, with growth rates determined by the average growth in each series from 2010-2020. This latter period of the transition path provides a meaningful window into concerns about Baumol-style stagnation, as constant growth in each TFP series implies that changes in aggregate growth primarily reflect the effect of ongoing structural change in the economy. To further focus on the limiting growth dynamics implied by the CES technologies, we discard the first 10 years of this window, in which GDP per worker growth partially reflects delayed capital accumulation dynamics from changes in TFP observed during the end of our data window.

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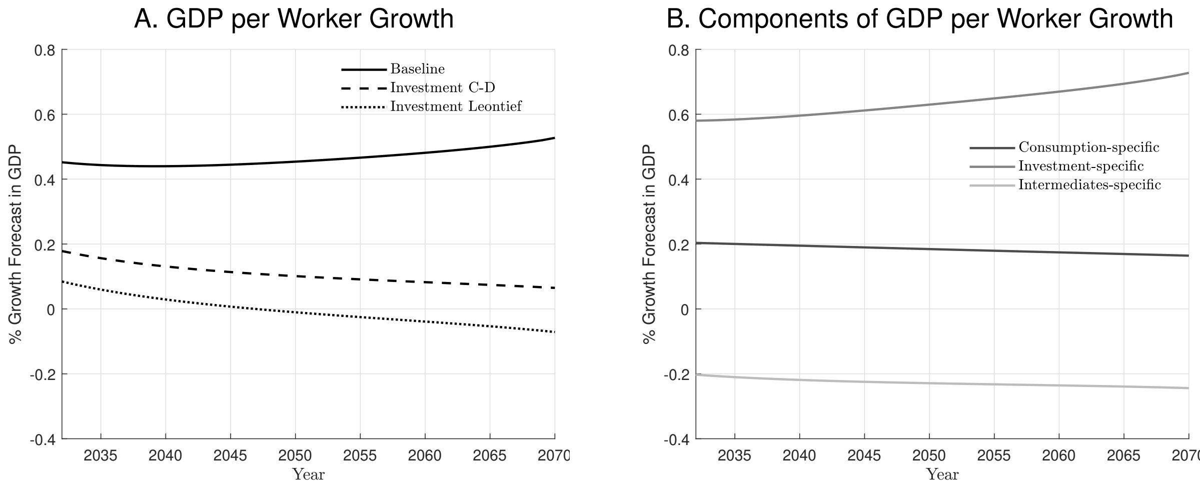
eliminating complementarity in consumption are roughly 40-45% as big as our baseline, but all other results are similar.

Figure 9: GDP Growth Forecasts and Counterfactuals, 2032-2070



Notes: Panel A shows the annual growth rate (in %) of aggregate GDP per worker in the model simulation from 2032-2070, where all TFP series grow at a constant rate; panel B shows counterfactual evolutions of the growth rate of GDP per worker over this period for three cases: (1) only TFP growth among consumption producers ("Consumption-specific"). (1) only TFP growth among investment producers and in investment-bundling ("Investment-specific"), and (3) only TFP growth in intermediates, both at intermediates producers and from intermediates bundling technical change ("Intermediates-specific"). Panel A also includes counterfactual GDP growth forecasts for the case where investment aggregation in each sector is Cobb-Douglas and where investment aggregation in each sector is Leontief.

In Panel A of Figure 9, we plot the predicted annual growth rate (in percent) of GDP per worker from 2032 to 2070 (solid line). Growth in GDP per worker is low in our forecast, starting at 0.45%, reflecting the low observed growth in sectoral TFP from 2010-2020. However, the figure shows that GDP gradually accelerates from 2032 to 2070, rising to 0.53% in 2070.

Panel B illustrates the channels underlying this growth forecast by plotting the growth rate of GDP per worker in counterfactual transition paths where we allow for TFP growth only for consumption producers, investment producers (and bundling technologies), or intermediates producers (and bundling technologies). Consistent with Baumol's cost disease, we see that when TFP growth occurs only in consumption or intermediates, aggregate growth slows over time, with reallocation to the slowest growing producers. However, because of substitutable inputs in investment, when there is only TFP growth at investment producers, growth is rising over time.

Finally, panel A of Figure 9 also presents counterfactual scenarios where the elasticity of substitution in investment inputs is set to either 1 (Cobb-Douglas) or 0 (Leontief)—assumptions commonly made in the literature. Absent endogenous reallocation within investment, aggregate growth is substantially lower and gradually converges toward zero (or even negative values, due to negative

TFP growth in intermediate sectors). These counterfactuals demonstrate the strength of Baumol's cost disease in standard growth models. Our baseline simulation, which shows rising aggregate growth in GDP per worker, reveals that investment reallocation serves as a critical counterbalance to the forces driving long-run stagnation from Baumol's cost disease. Indeed, this mechanism appears sufficient to fully offset the growth drag caused by structural change in consumption and intermediate production.

7. Conclusion

We document that services sectors have produced a larger share of both intermediates and investment over time. To understand these patterns, we develop a framework that allows us to study structural change in both consumption and production networks. Explicitly modeling intermediates allows us to use expenditure-side prices on final commodities, rather than income-side sectoral gross output prices, to discipline the model in a way that is internally consistent (as discussed by Herrendorf et al., 2014). Using novel price series disaggregated by both sector and final use, we find that goods and services are substitutes in the production of investment, rather than complements, as found in previous studies. We show that aggregation bias accounts for differences in these findings, with expenditure-side prices able to capture price growth in the production of investment that is easily averaged out in gross output prices.

This finding relates to a concern of Baumol (1967)—that structural change leads to a systematic reallocation from productive/innovative goods sectors to less innovative services sectors, eventually leading to an economy where the least productive sector dictates all economic progress. While the intermediates network in our framework does indeed appear to suffer from Baumol's "cost disease", we find the investment network to be the primary engine of growth in part because it is systematically shifting toward the most productive *services* (e.g., software development and R&D). Our findings thus provide some optimism for the impact of sectoral reallocation on aggregate economic growth.

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Appendix A. Data Sources and Details for Production Networks and Price Measurement

Appendix A.1. Measuring Production Networks

Our primary data source for measuring production networks and related data is the BEA Input-Output (IO) database, specifically, the time series of Make and Use tables from 1947-2020 and the time series of the investment network generated by vom Lehn and Winberry (2022) (henceforth vLW).[30] These data cover 40 NAICS-defined economic sectors, including agriculture and government (43 if energy/oil-intensive sectors are included); Table A.1 lists the 40 sectors and corresponding NAICS codes. More recent vintages of the IO database allow for greater sectoral detail, allowing the construction of more detailed investment networks. Given our interest in structural change over the long run, we focus on data 40 sectors that are consistently defined for the entire sample of 1947-2020.

The core of the Use table is a square matrix that reports intermediate input expenditures by different sectors (organized along columns) on specific commodities (organized along rows). These commodities are named and assigned NAICS codes based on which sectors are major producers of the given commodity, but more than one sector may be involved in the production of a given commodity. The mix of sectors that produce a given amount of each commodity is observed in the Make table, which is a square matrix reporting commodities along columns and the amounts of each commodity produced by each sector along rows. The final input-output matrix in each year is the matrix product of a scaled Make table, where each column is scaled by its sum (thus summing to 1) and the unscaled Use table.

The data from vLW reports the matrix of sectoral spending and production of new investment. Following their procedures, we extend the investment network data through 2020. Since vLW's investment matrix reports commodities rather than sectors in each row, we adjust their data using the the Make matrix. Our final investment network is represented by the product of the scaled Make matrix and the unscaled, extended investment network from vLW for 1947-2020.

For reference, we provide an illustration of the average structure of production networks in the United States from 1947-2020. For visualization, it is convenient to present scaled versions of

[30]Data are available at https://www.bea.gov/industry/input-output-accounts-data and https://doi.org/10.7910/DVN/CALDHX.

Table A.1: Average Share of Intermediates and Investment Production: Avg. 1947-2019

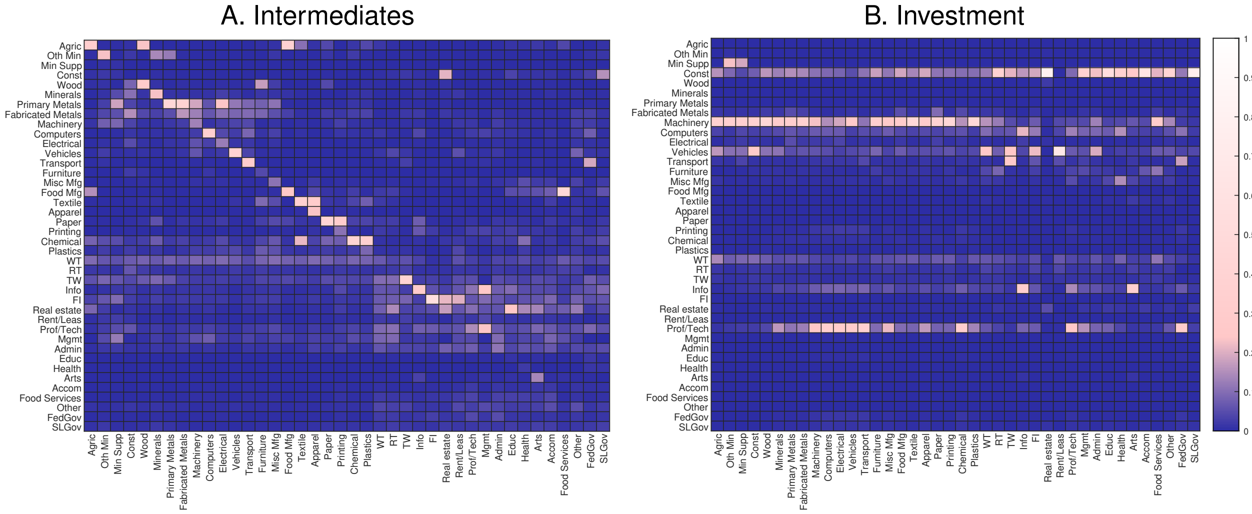
Goods Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.	Services Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.
Agriculture, forestry, fishing and hunting (11)	5.8	0.0	Wholesale trade (42)	5.8	3.8
Mining, except oil and gas (212)	1.1	0.2	Retail trade (44-45)	1.9	1.4
Support activities for mining (213)	0.0	0.3	Transport and warehousing (48-49, minus 491)	5.6	0.9
Construction (23)	1.9	37.0	Information (51)	4.6	6.2
Wood products (321)	1.5	0.5	Finance and insurance (52)	7.8	0.1
Non-metallic mineral products (327)	1.6	0.1	Real estate (531)	4.9	1.8
Primary metals (331)	4.8	0.1	Rental and leasing services (532-533)	1.1	0.0
Fabricated metal products (332)	4.2	1.1	Professional and technical services (54)	5.4	10.3
Machinery (333)	1.7	8.6	Management of companies and enterprises (55)	3.1	0.1
Computer and electronic products (334)	2.5	6.7	Administrative support and waste services (56)	3.3	0.1
Electrical equipment manufacturing (335)	1.2	1.0	Educational services (61)	0.3	0.2
Motor vehicles manufacturing (3361-3363)	3.3	9.1	Health services (62)	0.3	0.1
Other transportation equipment (3364-3369)	1.5	4.7	Arts, entertainment and recreation services (71)	0.4	0.1
Furniture and related manufacturing (337)	0.3	1.2	Accommodation services (721)	0.5	0.0
Misc. manufacturing (339)	0.9	1.3	Food services (722)	1.0	0.0
Food and beverage manufacturing (311-312)	4.9	0.1	Other private services (81)	1.7	0.1
Textile manufacturing (313-314)	2.0	0.3	Federal government (n/a, but incl. 491)	1.1	0.5
Apparel manufacturing (315-316)	0.7	0.0	State and local government (n/a)	1.4	0.9
Paper manufacturing (322)	2.6	0.0			
Printing products manufacturing (323)	1.2	0.6			
Chemical manufacturing (325)	4.6	0.4			
Plastics and rubber products (326)	1.8	0.1			

Notes: The table reports the average share of total intermediate and investment production by 40 consistent sectors. Individual components may not exactly sum to totals due to rounding. Sectors are classified according to the NAICS-based BEA codes, with 2007 NAICS codes in parentheses. Government sectors as defined by the BEA do not have naturally corresponding NAICS codes.

the two networks, with elements s^M_{ijt} and s^X_{ijt} representing sector j's share of total expenditures on intermediates (M) or investment (X) made by sector i in year t.

Figure A.1 plots heatmaps of the scaled input-output network (panel A) and the scaled investment network (panel B), averaged over the entire sample period. These heatmaps show the sparsity of both the input-output and investment networks; for any given sector, the majority of investment and intermediates are purchased from a small set of sectors. For the investment network, the distribution of investment producers is fairly similar across sectors. Most sectors purchase investment goods from a collection of prominent investment hubs—construction (structures), machinery and vehicles manufacturing (equipment), and information and professional/technical services (intellectual property). However, for the input-output network, there is much more sector-specificity as to which sectors are important suppliers of intermediates. In particular, we observe significant homophily in the input-output network—goods sectors play a large role as intermediates suppliers for goods sectors and services sectors play a large role as intermediates suppliers for services sectors.

Figure A.1: Heatmaps of Average Scaled Production Networks, 1947-2020



Notes: Panel A shows the scaled input-output network for intermediate goods; panel B shows the scaled investment network for new capital goods. Each (i, j) element shows the fraction of total expenditure by sector j (columns) from producing sector i (rows).

The BEA Use tables also contain data on the final uses of each commodity, including consumption. We measure structural change in consumption by constructing final consumption produced by each sector as the product of the scaled Make matrix and the sum of private final consumption and government consumption vectors in the Use table. While the Use table has information on the final use of each commodity as new investment, we use sums of the data from the investment network to compute total production of investment by each sector.

Appendix A.2. Data Sources for Price Measurement

We measure the price of consumption and the price of investment produced by goods and services sectors using expenditure-side accounting data. We start with final expenditure data (prices and total expenditures) for 68 consumption commodities (NIPA Tables 2.4.4 and 2.4.5) and 30 investment commodities (NIPA Tables 5.3.4, 5.3.5, 5.5.4, 5.5.5, 5.6.4). The 68 consumption commodities are the finest disaggregation available for the postwar period (after omitting commodities produced primarily by the energy-intensive sectors omitted from our analysis: motor vehicle fuels, fuel oils and other fuels, water supply and sanitation services, electricity, and natural gas). The 30 investment commodities (2 structures, 24 equipment, 3 intellectual property, and residential commodities) are a subset of the 33 investment commodities used in vLW. Due to data limitations, primarily on detailed investment prices not studied by vLW, we combine light trucks and other trucks into a single commodity, all software into a single commodity, and residential structures and residential equipment into a single commodity.[31]

With this mapping, we measure price growth for goods-consumption, services-consumption, goods-investment, and services-investment as a Tornqvist index as described in Section 3. The weights in that index depend on the sector's production share for each commodity, constructed from the BEA's published bridge files for consumption commodities[32] and the bridge files constructed by vLW, which we extend forward through the year 2020 using the latest BEA bridge files for investment. Both of these bridge files are multiplied by the scaled Make file. Since we focus primarily on the differences between goods and services production, we aggregate the final bridge files to two sectors, goods and services (as defined in Table A.1).

We also modify the bridge files for investment from vLW by assuming that goods produce the entirety of structures (including residential investment). In the original bridge files from vLW, a small fraction of structures (roughly 7% of non-mining structures and 11% of residential investment) is produced by services, primarily margin contributions from sectors like real estate and finance. However, because overall investment production by services is low compared to goods, especially early in the sample (as seen in Figure 2) and because investment spending on structures is large, absent any adjustment, a sizable portion of the services investment price is determined by structures prices, although the amount of total services investment production coming from structures is falling over time, from 1/3 in 1947 to less than 10% by 2020. This is unappealing because 1) it is unlikely that the price of structures investment reflects the price of these margins and 2) because the exact contribution of these services margins is determined in part by imputation (see Appendix A of vLW for further details). Thus, we set the services contribution to the production of structures equal to zero in our final bridge files. However, even without this adjustment, Appendix C documents that services investment prices are still significantly decreasing relative to goods.

[31]The 33 investment commodities in vLW are approximately the finest level of disaggregation of investment commodities possible while accurately tracking each commodity's production by different sectors over time. It is possible to study patterns in more finely disaggregated non-mining structures, but the mix of sectors producing these structures is the same for each commodity. Thus, further disaggregation does not yield additional insight. For this case, we aggregate prices for detailed structures to a single investment price for non-mining structures using a Tornqvist index.

[32]Available at https://www.bea.gov/products/industry-economic-accounts/underlying-estimates

We measure intermediates prices using the procedure described in Section 3. Gross output prices by sector are implicitly an average of the price of all commodities produced by that sector. Thus, using the price of consumption and investment produced by goods or services, we identify the price of intermediates produced by goods or services as the residual in gross output prices.

For all prices, we make adjustments for the omission of oil/gas prices; these adjustments are described in Appendix C.

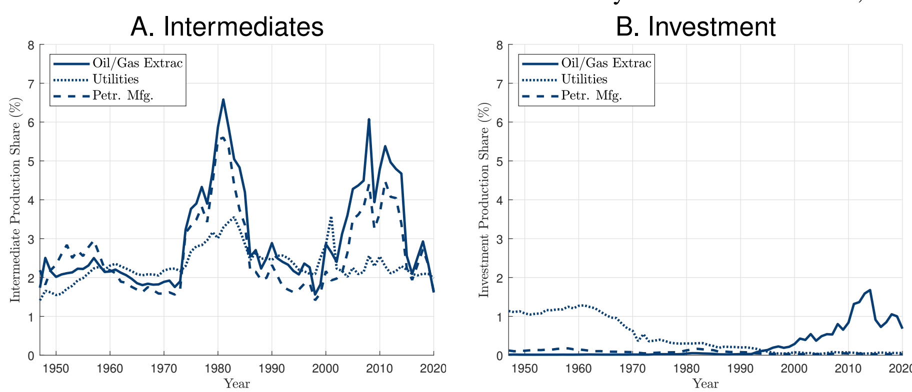
APPENDIX (FOR ONLINE PUBLICATION)

Appendix B. Additional Empirical Results

Appendix B.1. Energy/Oil-Intensive Sectors' Contributions to Structural Change

Figure B.2 plots the share of intermediates or investment produced by sectors omitted from our analysis – oil/gas extraction, utilities, and petroleum manufacturing. Although there are medium-run swings in these shares (particularly for intermediates), there is no long-run trend in how much these sectors produce of intermediates and only very slight (and off-setting) long-run trends in investment. Thus, as can be seen in Figure B.3, excluding these sectors does not significantly impact the long-run structural change trends observed in consumption, intermediates, and investment.

Figure B.2: Shares of Intermediates and Investment Produced by Oil-Intensive Sectors, 1947-2020



Notes: Panel A: share of intermediates produced by oil-intensive sectors; panel B: share of investment produced by these sectors.

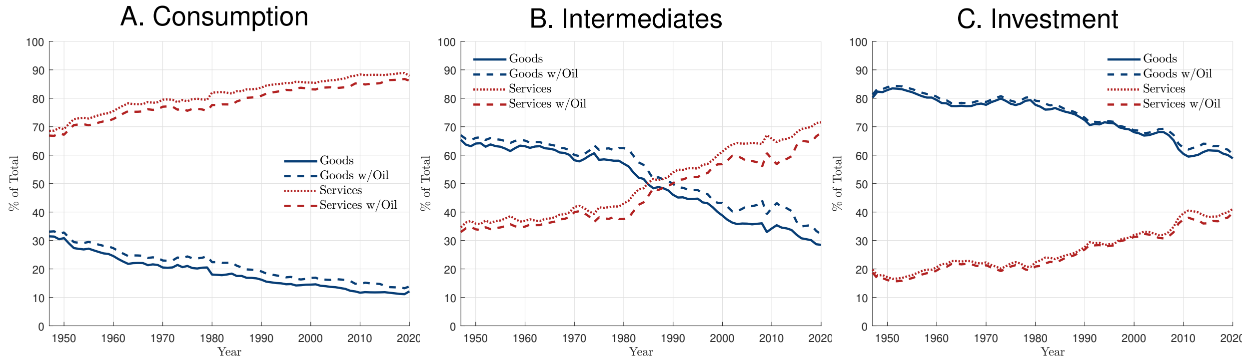
Appendix B.2. Time Series Detail for Rising and Falling Intermediates/Investment Producers

The four panels of Figure B.4 report the time series for sectors whose share of production of intermediates or investment has increased the most (right panels) or decreased the most (left panels). Each sector's changes in production shares appear to be part of a gradual long-run trend and not a single spike occurring in a particular year.

Appendix B.3. Within-Sector Changes in the Services Share of Intermediates and Investment

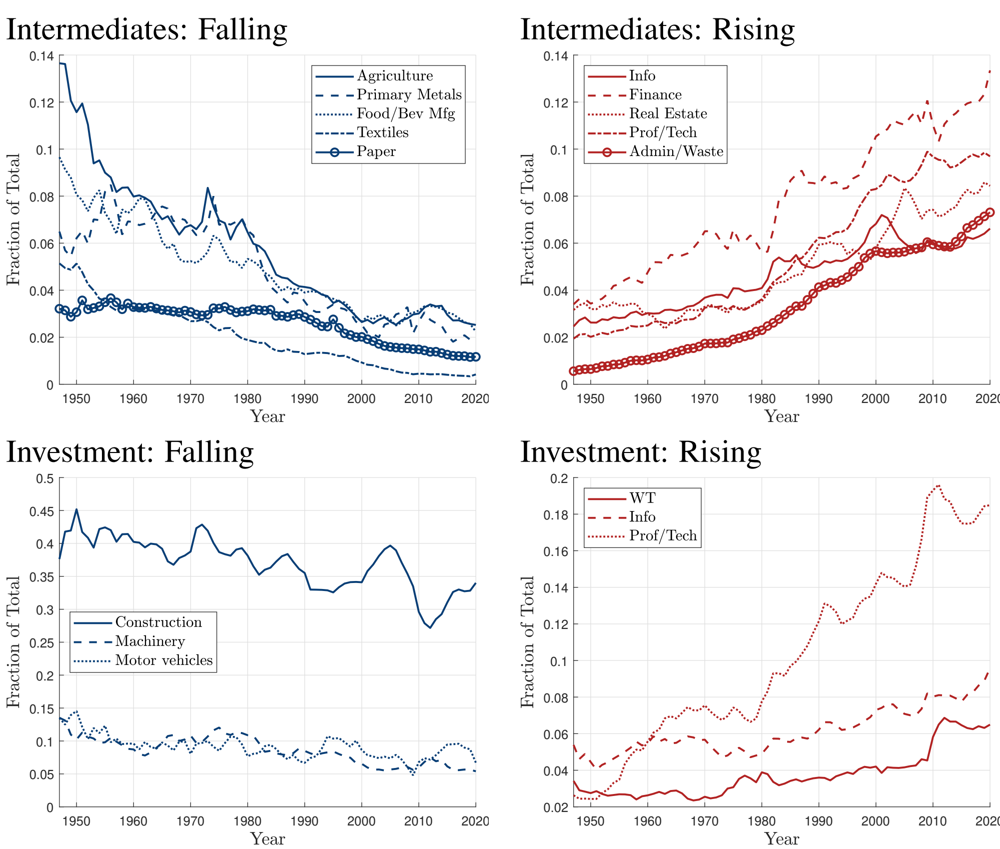
In Figure B.5, we plot bar charts showing the change in the share of intermediates (left panel) and investment (right panel) produced by services within all 40 sectors between 1947 and 2019. For both intermediates and investment, the share of production coming from services is increasing in almost every sector.

Figure B.3: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, With and Without Oil-Intensive Sectors, 1947-2020



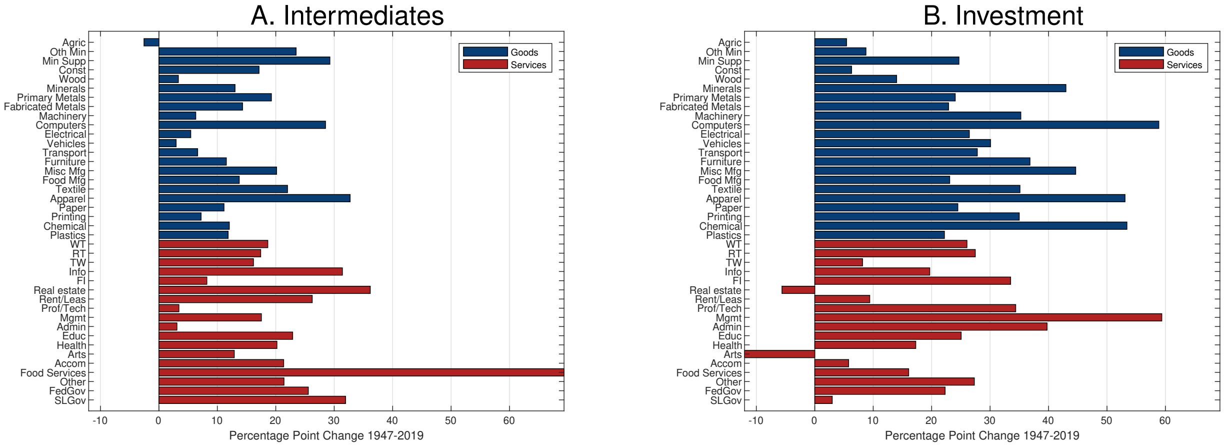
Notes: The figures plot the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue) and the services sector (red). Dashed lines indicate these series when oil-intensive sectors are included (all part of the goods sector).

Figure B.4: Time Series Changes in Production Share of Intermediates and Investment, Additional Sector Detail



Notes: Each line represents a given sector's share of total production of intermediates (top rows) or investment (bottom rows). Right panels with red lines show sectors whose production share has increased the most; left panels with blue lines show sectors whose production share has decreased the most.

Figure B.5: Changes in Services Production Share of Intermediates and Investment Purchased by Each Sector: 1947-2019



Notes: Each bar represents the change in the services production share of intermediates (left panel) or investment (right panel) purchased by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

Appendix B.4. Shift-Share Decomposition

We consider a shift-share decomposition of the rising share of intermediates and investment produced by services sectors, isolating changes occurring within and between sectors. This decomposition can be expressed as:

$$\Delta Serv_t = \underbrace{\sum_j^N \left(\overline{\omega}_j \Delta Serv_{jt}\right)}_{\text{within}} + \underbrace{\sum_j^N \left(\overline{Serv}_j \Delta \omega_{jt}\right)}_{\text{between}},$$

where $\Delta x = x_{2019} - x_{1947}$ is the change in x and $\overline{x} = \frac{x_{2019}+x_{1947}}{2}$ is the average of x in the two periods 1947 and 2019. The results of this decomposition are presented in Table B.2.

For investment, the large majority (75-100%) of all changes over time in the shares of investment produced by services are due to changes within sectors; for intermediates, the within-sector component contributes roughly half of the change over time. This evidence accords with the production network patterns shown in Figure A.1, which showed more sector-specificity in intermediate suppliers than investment suppliers across sectors, consistent with a more significant role for between-sector changes in the services share of production.

Appendix B.5. Decomposing Value-Added Measures of Structural Change

As explained in Herrendorf et al. (2013), a value-added approach to measuring sectoral production of consumption and investment focuses not only on the set of sectors producing the final product but also on the network of sectors contributing intermediate inputs needed to produce the

Table B.2: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment

				2 Sector Decomp.		40 Sector Decomp.	
	1947	2019	Δ	within	between	within	between
Intermediates	0.35	0.71	0.37	0.19	0.17	0.18	0.19
				(53%)	(47%)	(49%)	(51%)
Investment	0.20	0.40	0.20	0.21	-0.01	0.15	0.05
				(104%)	(-4%)	(73%)	(27%)

Notes: The table reports the results of shift-share decompositions of the share of services production over time. Individual components may not exactly sum to totals due to rounding.

product. Thus, value-added measures of structural change implicitly include structural change in intermediates and structural change in final producers of consumption and investment.

Value-added vectors of sectoral production of consumption and investment (denoted in current dollars), $\mathbf{c^{VA}}$ and $\mathbf{x^{VA}}$, are constructed using input-output data using the following equations:

$$\mathbf{c^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}}\mathbf{c} \tag{B.1}$$

$$\mathbf{x^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}}\mathbf{x} \tag{B.2}$$

where $\mathbf{c}$ and $\mathbf{x}$ are vectors of final production of consumption and investment, respectively, by each sector, $\mathbf{I}$ is the identity matrix, $\mathbf{v}$ is a diagonal matrix of the share of value added in gross output in each sector, and $\mathbf{\Gamma}$ is a matrix of input-output relationships, where the $(i, j)th$ element of $\mathbf{\Gamma}$ is the ratio of intermediates purchased by sector j from sector i to the total gross output in sector j.[33] We use BEA Make and Use table data to construct all the elements of equations (B.1) and (B.2).

Structural change in consumption or investment value added can occur because of changes in $\mathbf{v}$, changes in $(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}}$ (capturing the input-output network), or changes in $\mathbf{c}$ or $\mathbf{x}$ (final producers of consumption or investment goods). We consider counterfactuals where we allow only one of these three components to vary over time and hold the two other components fixed at their values in the initial year of our data, 1947. Since the construction of consumption and investment value-added is non-additive, the contributions of each of these three terms will not necessarily sum to one.

Table B.3 presents the decomposition, highlighting the contribution of each of these three forces to the change in the share of consumption value-added and investment value-added produced by the services sector between 1947 and 2019. Changes in the input-output network account for 45-

[33] We first compute all of these objects at the 40 sector level and then analyze structural change at the two sector level, aggregated up from consumption value added and investment value-added constructed at the 40 sector level.

Table B.3: Decomposing Structural Change in Services Share of Value Added Measures of Consumption and Investment

Services Share of:	1947	2019	Δ	% of Total
Consumption Value Added	0.68	0.87	0.20	
Final Prod. only	0.68	0.81	0.13	69%
Input-Output only	0.68	0.77	0.09	48%
VA share only	0.68	0.65	-0.02	-11%
Investment Value Added	0.36	0.54	0.18	
Final Prod. only	0.36	0.50	0.14	75%
Input-Output only	0.36	0.46	0.09	52%
VA share only	0.36	0.33	-0.03	-19%

Notes: This table reports the share of value-added based consumption and investment produced by the services sector and how this changes over time due to changes in each component of the value-added measure (as seen in Equations (B.1) and (B.2)). Changes generated by each of the three components—final producers ("Final Prod. only"), the Total Requirements Matrix ("Input-Output only"), and value-added shares of gross output ("VA shares only")—are computed by holding fixed all other components at their values in 1947. The "% of total" column refers to the change in each component divided by the change in total consumption or investment value added. Because of the non-linear nature of the decomposition, the total of each component will not sum to the actual total.

55% of the rising share of services production of consumption and investment value added. This total contribution is potentially slightly inflated because the contribution of each component sums to more than the total change in the services share of consumption and investment value added. However, if we compute the contribution of changes in the input-output network as a fraction of the sum of changes in each component, input-output changes still account for 40-50% of structural change in consumption and investment value added.

Appendix B.6. Evidence on Outsourcing

A potential concern about the observed pattern of structural change in intermediates is that it may reflect outsourcing of services tasks, resulting in a change of *where* services tasks are performed and how they are recorded, rather than a change in the actual structure of production. That is, it may be that firms who originally produced services internally started outsourcing these tasks to other firms, with the newly outsourced tasks measured as services intermediates.

We provide three pieces of evidence to suggest that such outsourcing of services does not drive the observed structural change in intermediates. First, the national accounting data we use measures economic activity at the establishment level. Thus, services provided to establishments within a firm by separate administrative offices and headquarters are already classified as intermediates produced by services. If an establishment now receives these services from a different firm, it would not impact the measured share of intermediates produced by services.

Second, the average ratio of spending on intermediates relative to gross output (across sectors) has remained between 42-46% for nearly the entire sample window of 1947-2020, with little trend over time. Furthermore, within sectors, there is no correlation between increased intermediate spending (relative to gross output) and an increase in the share of intermediates purchased from services. Thus, structural change in the production of intermediates does not appear to have coincided with increased spending on intermediates.[34]

Finally, we extend an argument made by Duernecker and Herrendorf (2022) to analyze structural transformation of the occupational distribution. If outsourcing were the primary driver of structural change in intermediates, we would expect to see the following two patterns in the data. First, we would expect a decline in services-task intensive occupations within goods producing industries. Second, we would expect that changes in services task intensity within sectors should be negatively correlated with changes in purchases of services intermediate inputs.

To investigate this concern, we utilize data from decennial U.S. Censuses for the years 1950-2010 and the 2019 American Community Survey (both provided by IPUMS 13.0) to document trends in the industry-specific concentration of services-task intensive occupations. We consider services-task intensive occupations as (1) managerial/ professional/ specialty, (2) technical/ sales/ admin, and (3) services occupations based on the IPUMS 13.0 `OCC1990` classification.

Using the IPUMS 13.0 `IND1990` industry aggregation, we construct 32 consistent sectors over the period 1950-2019 following vom Lehn (2018), listed in Table B.4.[35] These sectors are a direct aggregation of the 40 NAICS sectors used in our main analysis and can therefore be directly compared over the entire sample. For each of these 32 sectors we construct two measures for the services-task intensity: the services employment share (total employment in services-task intensive occupations divided by total employment within the sector); and the services earnings share (total earnings of services-task intensive occupations divided by total earnings within the sector).

Table B.4 illustrates that employment and earnings shares of services intensive occupations

[34] A limitation of this argument is that the ratio of spending on intermediates to total gross output is not the cost share of intermediates in production due to the presence of markups. If markups rise over time, a constant ratio of intermediates spending could imply a rising cost share of intermediates, possibly reflecting increased outsourcing. The evidence on the markup trends is mixed (see Basu, 2019).

[35] IPUMS data does not allow us to distinguish the information services sector from the printing and publishing manufacturing sector throughout the postwar sample. We thus combine these two sectors into one and list it as a goods sector, though listing it as a services sector does not alter our results.

Table B.4: Change in Occupational Service Task Intensity Within Sectors, 1950-2019

Goods Producing Sectors (NAICS Codes)	Perc. Pt. Ch. Emp.	Perc. Pt. Ch. Earn.	Service Producing Sectors (NAICS Codes)	Perc. Pt. Ch. Emp.	Perc. Pt. Ch. Earn.
Ag./forestry/fishing/hunting (11)	-7.0	-8.6	Wholesale trade (42)	4.7	16.0
Mining, except oil and gas (212)	16.6	25.0	Retail trade (44-45)	5.7	9.2
Construction (23)	11.4	24.0	Transport and warehousing (48-49, minus 491)	9.7	20.2
Wood products (321)	9.3	21.3	Finance and insurance (52)	1.0	1.3
Non-metallic mineral products (327)	15.8	31.5	Real estate (531)	3.4	6.3
Primary and Fabricated Metals (331,332)	13.4	25.4	Prof./Tech./Rent./Mgmt/Admin. (54-56,532-533)	8.2	12.0
Machinery (333)	16.4	31.1	Educational services (61)	1.6	2.9
Computer and Electronic Mfg (335,334)	35.8	53.8	Health services (62)	2.3	4.2
Motor Vehicles Mfg (3361-3363)	12.9	30.3	Arts, ent. and rec. services (71)	5.3	4.4
Other transp. equipment (3364-3369)	28.9	43.8	Accommodation services (721)	1.5	3.3
Furniture and related MfG (337)	13.9	30.2	Food services (722)	-2.5	-2.4
Misc. manufacturing (339)	28.9	49.2	Other private services (81)	11.9	14.1
Food and Beverage Mfg (311-312)	8.0	23.3	Fed/State/Local Government (n/a, but incl. 491)	12.0	13.8
Textile manufacturing (313-314)	20.1	37.0			
Apparel manufacturing (315-316)	24.7	47.1			
Paper manufacturing (322)	11.5	25.4			
Printing and Information (51,323)	16.2	23.1			
Chemical manufacturing (325)	21.2	37.7			
Plastics and rubber products (326)	8.8	25.8			

Notes: The table reports the percentage point change in occupational employment and earnings shares bteween 1950 and 2019 within 32 consistent sectors. The sectors are a direct aggregation of the 40 NAICS 2007 sectors listed in Table A.1 to map into the IPUMS IND1990 classification as suggested by vom Lehn (2018). Services intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, and (3) services occupations based on the IPUMS 13.0 OCC1990 classification. Data on employment are taken from the U.S. Census obtained from IPUMS USA.

within all but two (agriculture, and food services) of the 32 broad NAICS sectors have substantively increased over the period 1950-2019. This implies that the structural change patterns we document for the intermediates network do not coincide with a reduction in services workers in goods sectors.

We also find that changes in occupational services-task intensity are not negatively correlated with changes in the purchases of intermediates produced by services sectors. To show this, we construct changes in our two measures of within-sector service intensity by decade and correlate these with changes in the each sector's share of intermediates expenditures produced by services. We regress the change by decade in each measure of sectoral services task intensity on a constant, a full set of time effects (decade dummies), and the sectoral change by decade in the share of intermediates expenditures produced by services sectors. Table B.5 summarizes the regression estimates, finding no systemic correlation between within industry changes in the share of intermediates purchased from services and changes in service task intensity.

Table B.5: Changes in Service Task Intensity and Structural Change

	ΔService Task Intensity	
	Employment Share	Earnings Share
	(1)	(2)
Δ Share of Intermediates Expenditures from Services	2.036	2.806
	(3.207)	(4.680)
Obs.	224	224

Notes: The table reports results from linear regressions, where the left-hand side is the decadal change in the employment (column 1) or earnings share (column 2) of services-task intensive occupations within 32 consistent NAICS sectors (listed in Table B.4). The regressors are a constant, a full set of time effects (coefficients not reported), and the decadal change in the share of intermediates purchased from services sectors. Standard errors are reported in parentheses underneath the coefficients. The decadal changes are based on the years 1949, 1959, 1969, 1979, 1989, 1999, 2009, 2018. Data on employment and earnings are taken from the decadal U.S. Censuses for the years 1950-2010 and the ACS for 2019 (taken from IPUMS 13.0).Services intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, (3) service occupations based on the IPUMS 13.0 OCC1990 classification. Standard errors are reported in parentheses.

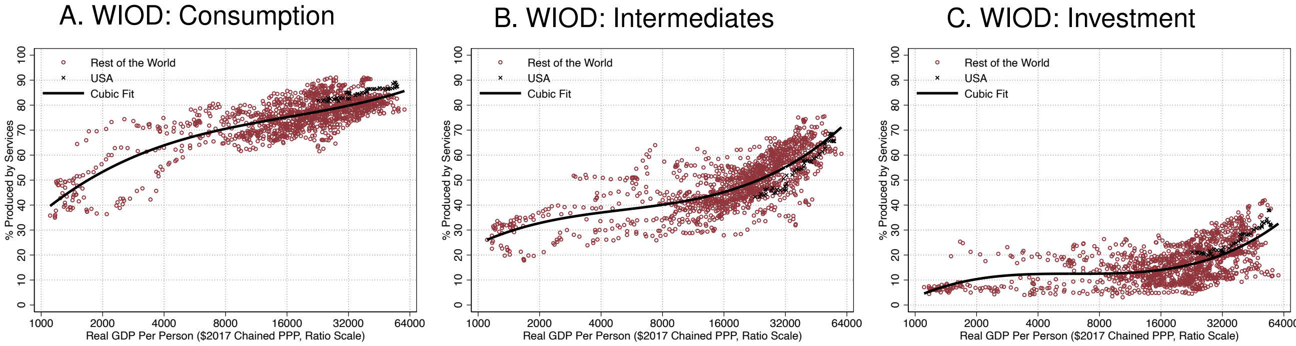
Appendix B.7. International Evidence

The broad patterns of structural transformation in the production of consumption, intermediates, and investment that we find in BEA data for the United States are also occurring in many countries around the world. To show this, we use two waves of the World Input Output Database (WIOD: Timmer et al., 2015; Woltjer et al., 2021) covering 41 countries (including the United States). The first wave reports data from 1965-2000 for 25 countries and 23 sectors, while the second covers data for 40 countries and 35 sectors over the period 1995-2011. Industries are classified according to the International Standard Industrial Classification revision 3 (ISIC Rev. 3) and we aggregate these industries to 21 consistently defined sectors that can be grouped into goods and services. As in our main analysis, we exclude oil and utilities producing industries.

To harmonize the two WIOD waves we measure the service/goods production shares starting with the level observed in the first year of the data and then construct shares in subsequent years by cumulating observed annual growth rates in these shares. For countries spanning both WIOD waves, we use the growth rates from the 2013 WIOD starting with the year 2000. We drop two countries: Hong Kong, because its data ends in 1999 and Luxembourg because its services share of investment is an outlier (about twice as large as that of countries at similar levels of development).

Figure B.6 plots structural change in consumption, investment, and intermediates using the

Figure B.6: International Trends in Goods/Service Production Share of Consumption, Intermediates and Investment (1965-2011)



Notes: Panels A-C display the share of services in the production of consumption, investment, and intermediates using the use tables from the World Input Output Database (WIOD, see Timmer et al., 2015; Woltjer et al., 2021) plotted against real GDP per capita (taken from the Penn World Table 10.0: Feenstra, Inklaar and Timmer, 2015). Panels A-C additionally show a fitted cubic polynomial and also highlight the data points for the USA (black Xs).

WIOD data. To facilitate comparison across countries, we plot the share of consumption, intermediates, and investment produced by the service sector against real GDP per capita (in $2017 chained PPP on a ratio scale), similar to illustrations provided by Galesi and Rachedi (2018). This figure reveals several notable insights. First, we highlight data for the United States (marked with black Xs) to illustrate that the range of values for the U.S. service shares constructed from WIOD data are very similar to those we observe from BEA data in Figure 2. Second, the fitted cubic trend lines suggest that the time series patterns observed in the United States are representative of the typical experience in other countries at similar levels of development (as measured by real GDP per capita). Third, increasing services shares in the production of consumption, intermediates, and investment is present at all levels of development within the WIOD database. It appears that structural transformation in the production of consumption is faster at lower levels of development, while that for intermediates and investment appears to accelerate at higher levels of development.

Appendix C. Additional Price Measurement Details and Robustness

Appendix C.1. Adjustment for Energy Prices

Although we omit sectors closely tied to market fluctuations in oil/energy prices from our analysis, these fluctuations may have a nontrivial impact on gross output prices via intermediate inputs. Because oil/energy is an intermediate input for many sectors, the final price of those sectors' output may reflect fluctuations in these prices. To abstract from fluctuations in the price of oil/energy in analyzing long-run price trends, we adjust gross output prices for the impact of oil/energy prices operating through intermediate input prices.

Consider the following representation of the evolution of gross output and intermediates bundle prices for sector j:

$$\Delta \ln(P_{jt}) = \alpha_{jt} \Delta \ln(P_{jt}^Y) + (1 - \alpha_{jt}) \Delta \ln(P_{jt}^M)$$
$$\Delta \ln(P_{jt}^M) = \sum_{i \notin E} s_{ijt}^M \Delta \ln(P_{it}) + \sum_{i \in E} s_{ijt}^M \Delta \ln(P_{it})$$

where P_{jt} is the gross output price of sector j, P_{jt}^Y is the (implicit) price of value added in sector j, P_{jt}^M is the price of the intermediates bundle purchased by sector j, and s_{ijt}^M represents elements of the input-output matrix for sector j at time t. The set E describes a set of sectors we wish to "exclude," yet still impact intermediate bundle prices and thus gross output prices in each sector.[36]

The following dynamics define the "adjusted" price series we seek to obtain:

$$\Delta \ln(\tilde{P}_{jt}) = \left(\alpha_{jt} + (1 - \alpha_{jt}) \sum_{i \notin E} s_{ijt}^M \right) \Delta \ln(P_{jt}^Y) + (1 - \alpha_{jt})(1 - \sum_{i \notin E} s_{ijt}^M) \Delta \ln(\tilde{P}_{jt}^M)$$
$$\Delta \ln(\tilde{P}_{jt}^M) = \sum_{i \notin E} \frac{s_{ijt}^M}{1 - \sum_{k \in E} s_{kjt}^M} \Delta \ln(\tilde{P}_{it})$$

These two linear systems can be solved using matrix algebra, implying that the adjusted gross output price series can be written as a function of the original gross output prices, expenditure shares and the prices of the excluded sectors:

$$\Delta \overrightarrow{ln(\tilde{P}_t)} \;\; = \Phi \left(\Delta \overrightarrow{ln(P_t)} - (I - diag(1 - \alpha_t) \Gamma_t')^{-1} diag(1 - \alpha_t) \left(\overrightarrow{s_t^{M,E}} \right)' \Delta \overrightarrow{P_t^E} \right)$$

where

$$\Phi = (I - diag(1 - \alpha_t) \Gamma_t')^{-1} diag \left(1 + \frac{(1 - \alpha_t) \sum_{i \notin E} s_{it}^M}{\alpha_t} \right) (I - diag(1 - \alpha_t) \Gamma_t'),$$

$\overrightarrow{ln(\tilde{P}_t)}$ is a vector of adjusted gross output prices, $\overrightarrow{ln(P_t)}$ is a vector of observed gross output prices, $diag(1 - \alpha_t)$ is a diagonal matrix with $1 - \alpha_{jt}$ along the diagonal, Γ_t is the input-output network with ij-th element s_{ijt}^M, $\overrightarrow{s_t^{M,E}}$ is a vector of the input-output network elements corresponding to

[36] The above representation only approximates how price measurement is done at the BEA for two reasons. First, the BEA uses Fisher indices instead of Tornqvist indices; however, the practical differences between these methodologies are negligible; the Tornqvist index is easier to analyze. Second, the above representation abstracts from imported intermediates, considering the gross output price of a sector j and an intermediate input made by that sector to be the same. However, in the case of gasoline and oil, global movements and domestic movements in prices are very similar, suggesting minimal bias from ignoring the potential for differential prices for imported oil.

excluded sectors, and $\overrightarrow{P_t^E}$ is a vector of prices of sectors to be excluded.

When inferring intermediates prices from gross output prices, we aggregate up these adjusted gross output prices across sectors to obtain adjusted goods and services gross output prices.

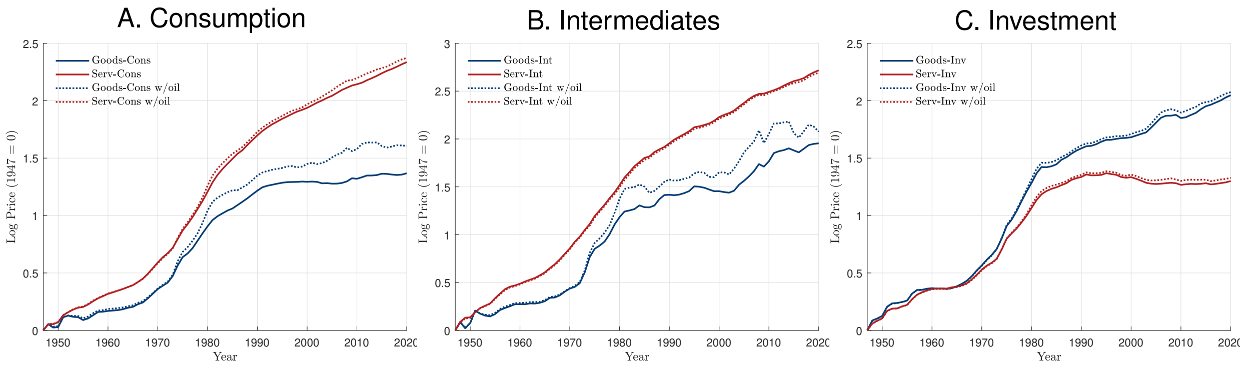
We also apply corrections to the consumption and investment prices using the difference between adjusted and unadjusted gross output prices. Given an "adjustment term," the difference between the adjusted price and the original gross output price ($\tilde{P}_{jt} - P_{jt}$), we adjust the final price of each commodity using a weighted sum of sectoral adjustment terms, weighted by each sector's position in the bridge/make files for each commodity. Formally, the final commodity prices for each consumption (or investment) commodity l are given by $\tilde{P}_{lt} = P_{lt} + \sum_{j=1}^{N} \xi_{jlt}^{C} \left(\tilde{P}_{jt} - P_{jt} \right)$, where ξ_{jlt}^{C} are the weights generated from the combination of the bridge and make files for commodity k (in the case of consumption). The impacts of this adjustment for consumption and investment prices are generally small, as can be seen below.

Figure C.1 plots the price of consumption, investment, and intermediates produced by goods and services sectors when oil/energy sectors are excluded and included. The impact of including oil sectors is greatest on goods sectors – the price of goods consumption rises more with the inclusion of energy-intensive commodities, and the price of goods intermediates is much more volatile. However, including oil sectors does not change the relative price patterns observed in the data and has a negligible impact on investment prices. The primary impact of including oil prices in our data would be to shrink the increase in the price of services-intermediates relative to goods-intermediates.

Appendix C.2. Comparison to PPI Prices

Although data from the PPI is ill-suited to our purposes, we can still compare our measured intermediates prices for the periods intermediate input prices are available from the PPI to validate our procedure. The PPI publishes the prices of four broad types of intermediate inputs: processed goods, unprocessed goods, services, and construction. To construct a goods intermediate price from these data, we aggregate processed goods and unprocessed goods. We abstract from construction intermediates prices because they comprise only about 1% of all intermediates, and data

Figure C.1: Sector Prices With and Without Oil Sectors Included



Notes: Solid lines denote baseline prices (as originally observed in Figure 3); dotted lines denote prices measured with oil/energy sectors included.

on construction intermediates are only available beginning in 2009.[37] As PPI data is available monthly, we average monthly prices to the annual frequency to compare with our annual data.

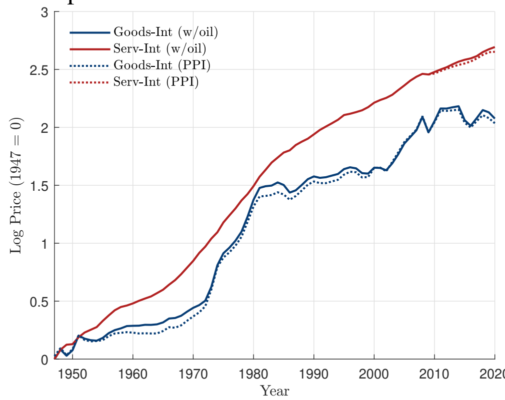
Figure C.2 compares our prices for goods and services intermediates with the intermediates prices constructed from PPI data. We use our price series without adjustments for oil/energy prices (including all oil/energy sectors) because applying our adjustment procedures to the PPI data is not feasible. For services intermediates prices from the PPI, we normalize prices in the first year they are available (2009) to match our services intermediates price series (for comparison). For both series, our inferred intermediates prices align closely with those published in the PPI.

Appendix C.3. Robustness of Relative Price Measurement

First, we consider the following three robustness exercises: 1) not omitting any sectors producing oil/energy products or making any adjustment for oil/energy price fluctuations, 2) reclassifying margin contributions (i.e., wholesale trade, retail trade, and transportation/warehousing margins) to production as goods sectors, and 3) setting bridge files to their average value throughout the entire sample (i.e., no time variation in bridge files). The relative prices of services to goods in consumption, investment, and intermediates in these three cases are compared to our baseline relative prices

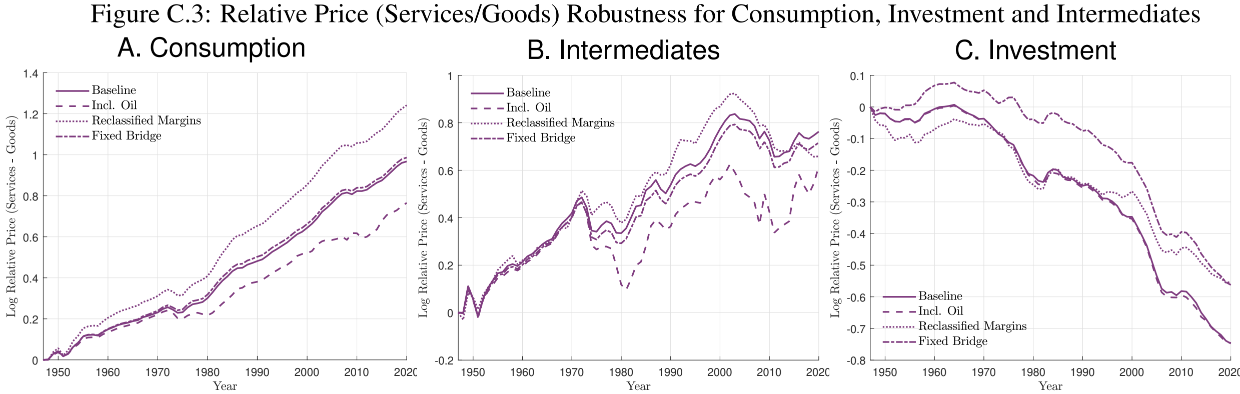
[37]The BLS does not publish aggregate weights that would facilitate aggregation of these price series to a single price series for goods (or all intermediates in general). However, in correspondence with economists at the BLS, we found out that the approximate weight on processed goods is 80%, and the approximate weight on unprocessed goods is 20%. We use these approximate weights to aggregate the two price series.

Figure C.2: Comparison of Measured Intermediates Prices to PPI Data



Notes: Solid lines denote measured prices including oil/energy sectors; dotted lines denote prices obtained from the Producer Price Index (PPI). PPI data on services intermediates prices are only available beginning in 2009.

Figure C.3: Relative Price (Services/Goods) Robustness for Consumption, Investment and Intermediates



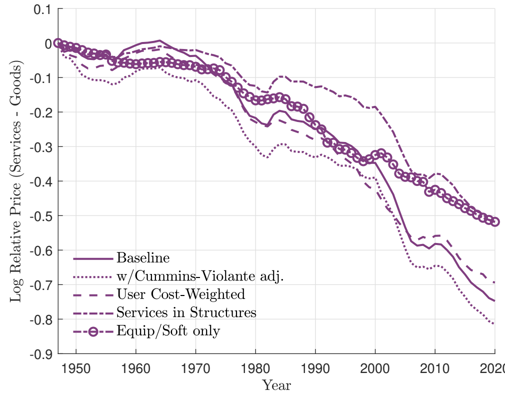
Notes: Solid lines denote baseline relative prices (as originally observed in the right panel Figure 3); dashed lines are prices inclusive of all oil/energy sectors (excluded in the baseline), dotted lines denote prices where margin sectors have been reclassified as goods sectors, and dash-dotted lines indicate prices measured when the bridge file mapping production of each commodity into production sectors is held fixed at its average value across the years 1947-2020.

in Figure C.3. In each case, the relative price series are similar to our baseline measures.[38]

Second, we consider four robustness checks focused on the relative price of investment: 1) using user cost weights to aggregate investment prices (i.e., rental services measures, constructed originally in vLW and extended through the year 2000), 2) adjusting investment prices to correct for quality bias, based on an extension to Cummins and Violante (2002), 3) allowing for structures to be partially produced by services sectors (as discussed in Appendix A), and 4) focusing exclusively

[38]The slightly different dynamics in the relative price of investment when holding bridge files fixed are largely attributable to changes in the importance of various margin sectors over time. If we both hold bridge files fixed and reclassify margins sectors, there is little impact on the relative price of investment from holding the bridge files fixed.

Figure C.4: Robustness of Investment Price Measurement



Notes: The solid line denotes the baseline relative price of investment (services/goods); the dotted line is the relative price with quality adjustments to equipment prices as presented in Cummins and Violante (2002), the dashed line is the relative price where prices are aggregated with user cost of capital weights, the dash-dotted line is the relative price when services are allowed to contribute to the production of structures, and the line with circle markers is the relative price when we only consider equipment and software investment prices.

on investment prices for equipment and software.[39] We plot the relative price of services investment to goods investment in each of these four cases and our baseline results in Figure C.4. In each case, the relative price of services investment falls relative to the price of goods investment.[40]

Appendix C.4. Comparison to Gross Output Price Measurement

As discussed in Section 3, an alternative way to measure consumption, investment, and intermediates prices is to use gross output prices. The simplest approach is to construct a single price for goods and a single price for services and assume this price applies equally to consumption, investment, and intermediates products. This parsimonious approach is used by Herrendorf et al. (2021), García-Santana et al. (2021), and Sposi et al. (2021) among others.[41]

With a single price for goods sectors and a single price for services sectors, a long literature has documented that the relative price of services is rising over time. This is unsurprising given our findings in Figure 3. Services prices are rising significantly faster than goods prices in intermediates and consumption, and these two uses make up the majority of gross output. Thus, aggregating

[39]We extend the quality adjustments in Cummins and Violante (2002) by applying the percent difference between the quality-adjusted prices of each commodity and the actual prices from 1995-2000 to all years since 2000.

[40]Even if we focus only on equipment investment produced by goods and services, the relative price of investment produced by services still falls, including when reclassifying margin sectors as goods sectors.

[41]Herrendorf et al. (2021) measure the prices of goods and services using a "value-added" approach, embedding the input-output structure of the economy in how services and goods prices are aggregated. However, the overall trends in relative prices are qualitatively similar when not making this adjustment.

the prices of consumption, investment, and intermediates into a single price masks heterogeneity in the price trends across different final uses.

It is also possible to obtain consumption-, investment- and intermediates-specific prices for goods and services just using gross output data. Using input-output data, we can observe how each subsector within goods and services contributes to producing consumption, investment, and intermediates. We can then aggregate up sectoral gross output prices in proportion to how much each subsector produces of consumption, investment, and intermediates. Formally, the price growth in use $u \in C, X, M$ produced by sector $j \in g, s$ can be measured from gross output data as:

$$\Delta \ln(P^u_{jt}) = \sum_{i \in j} s^{u,j}_{it} \Delta \ln(P_{it}) \tag{C.1}$$

where P^u_{jt} is price of use u produced by sector j at time t, i denotes the individual subsectors within j, $s^{u,j}_{it}$ is the share of all of sector j's production of use u that is produced by subsector i (averaged between time periods t and $t-1$), and P_{it} is the gross output price of sector i.

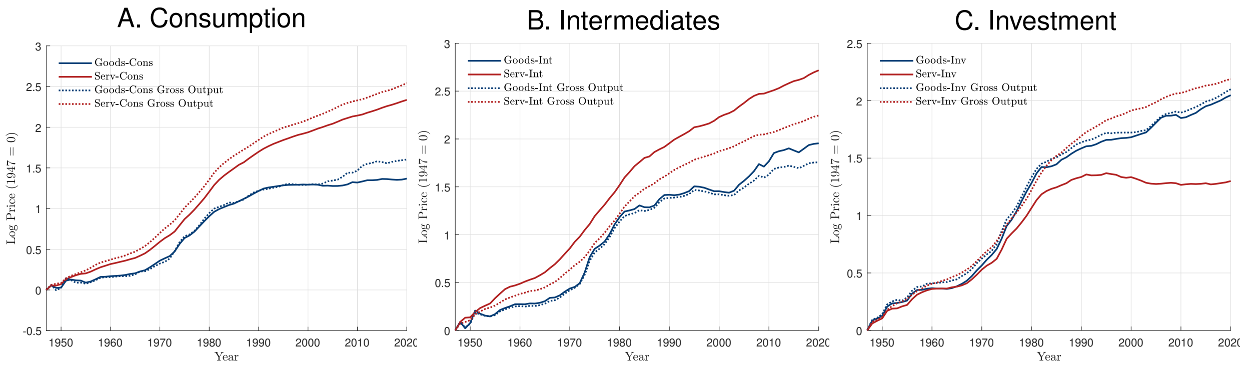
With this approach, variation in goods or services prices across consumption, investment, and intermediates is driven by heterogeneity in which subsectors produce each final product and heterogeneity in price growth in those subsectors. In principle, this gross output procedure could exactly replicate the prices we construct using expenditure side data if each detailed subsector only produces one of consumption, investment, and intermediates or if each of those products has the same final price. The equivalence of this gross output measurement approach and our expenditure-side measurement approach ultimately depends on how disaggregated the source data is.

Figure C.5 plots the consumption, investment, and intermediates prices for goods and services sectors constructed using gross output data according to equation (C.1).[42] These gross output prices are noticeably different than those we construct using expenditure-side accounting data, particularly for investment. Although price growth in services-investment is clearly lower than price growth in other services sectors (and thus services as a whole), the services-investment price is rising over time relative to the goods-investment price, the opposite of our baseline findings.

However, measuring the price of services-investment using gross output prices is likely subject

[42] We use the oil/energy price adjusted gross output prices for consistency with how we measure our baseline prices.

Figure C.5: Constructing Sector Prices Using Gross Output Data



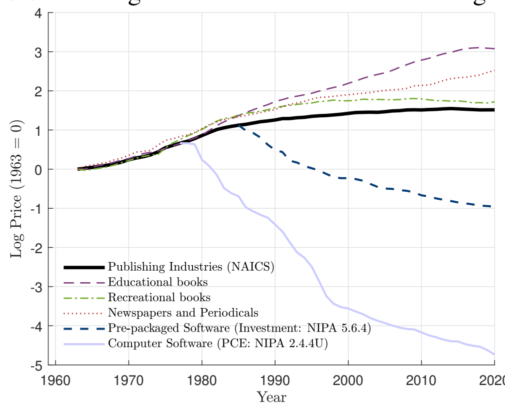
Notes: Solid lines denote baseline prices; dotted lines denote prices constructed using gross output data according to equation (C.1).

to significant aggregation bias, even when drawing on the disaggregated 40 sectors in Table A.1. For the two services sectors that produce the most services investment (information and professional/technical services) only a small portion of gross output is used as investment (on average only 15% and 22%, respectively). Moreover, as seen in Section 3, there is substantial price heterogeneity in the commodities produced by the professional/technical services sector, implying that using a single gross output price for this sector will overstate price growth in services-investment.

Although more recent data allows for greater disaggregated detail in BEA gross output prices – 62 non-oil/gas sectors since 1963, including greater detail within both the professional/technical services and information sectors – this level of disaggregation does not resolve the core issues of aggregation bias. Seven of the 39 services sectors still produce roughly 80% of all services investment.[43] Those seven sectors, listed in descending order of importance for producing services-investment (averaged over 1963-2020) are miscellaneous professional/technical services, wholesale trade, computer systems design and related services, broadcasting and telecommunications, publishing industries, retail trade, and motion picture and sound recording industries. However, despite this added detail in sectors, investment remains a small fraction of the uses of these sectors' gross output. For those 7 sectors, respectively, the share of output used for investment (averaged over 1963-2020) is 24%, 10%, 49%, 10%, 23%, 3%, and 29%. Thus, the added disaggregation in

[43] We omit the real estate margin contributions to investment, most of which are omitted in the construction of the investment network in vLW. Much of the remaining 20% of investment represents R&D expenditures produced within sectors, and in all of these sectors, the share of gross output used for investment is less than 5% on average, suggesting that gross output prices are unlikely to accurately represent the price of investment made by these sectors.

Figure C.6: Heterogeneous Prices Within Publishing Industries



Notes: The thick line denotes the gross output price for the NAICS sector "publishing industries", obtained from the BEA GDP by Industry database. The thin lines are the prices of commodities produced by the publishing sector, based on NIPA Tables 2.4.4U (Personal Consumption Expenditures) and 5.6.4 (Investment).

more recent data does not generate sectors that specialize in investment, which is needed to reduce the possibility of aggregation bias.[44]

As an additional example of price heterogeneity within this 62-sector level of disaggregation, consider the publishing industries sector (within the information sector). This sector produces investment in the form of pre-packaged software, but also produces many consumption products: consumer software, recreational books, educational books, and newspapers and periodicals. In Figure C.6, we plot the evolution of prices for this subsector and the commodities it produces.[45] Although the gross output price of publishing industries is not growing very much over time, the price of pre-packaged software is substantially *falling* over time. However, because the price of newspapers and varied books are rising substantially, aggregation to a single price for this sector fails to capture the price of investment it produces.

The above exercises, coupled with those presented in Section 3 illustrate the limitations of using gross output prices for generating prices by both production sector and product use.

Appendix D. Equilibrium Conditions and Proofs

[44] We find, in results available upon request, that using more disaggregated sectors to measure the price of services-investment from gross output data generates a slower growing price series than what is plotted in Figure C.5, and is thus closer to our prices measured using expenditure side data. However, the evidence presented here explains why these more disaggregated data do not yet generate the exact same services-investment price as in our baseline using expenditure side data.

[45] Consumer computer software and pre-packaged software prices are only available since 1977 and 1985.

Appendix D.1. Equilibrium Conditions and Derivations

The household's problem is

$$\max_{C_{jt},K_{jt+1}} \sum_{t=0}^{\infty} \beta^t log\left(\left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C-1}}\right),$$

$$\text{s.t. } \sum_j P_{jt}C_{jt} + \sum_{j=1}^{N} P_{jt}^X\left(K_{jt+1} - (1-\delta_j)K_{jt}\right) \leq W_t + \sum_j R_{jt}K_{jt}.$$

The first order conditions for this problem, $\forall j$, are

$$\frac{P_{jt}^X}{E_t^C} = \frac{\beta}{E_{t+1}^C}\left(R_{jt+1} + P_{jt+1}^X(1-\delta_j)\right) \tag{D.1}$$

$$\frac{P_{jt}}{E_t^C} = \left(\omega_{Cj}\frac{C_t}{C_{jt}}\right)^{\frac{1}{\epsilon_C}}\frac{1}{C_t} \tag{D.2}$$

where total consumption, C_t, and total expenditures (denoted in units of the numeraire), E_t^C, are given by:

$$C_t \equiv \left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C-1}} \tag{D.3}$$

$$E_t^C = \sum_j P_{jt}C_{jt}. \tag{D.4}$$

The profit maximization problem for the representative production firm in sector j is given by

$$\max_{L_{jt},K_{jt},M_{jt}} P_{jt}Q_{jt} - W_tL_{jt} - R_{jt}K_{jt} - P_{jt}^M M_{jt}, \text{ s.t. } Q_{jt} = A_{jt}\left(K_{jt}^{\theta_j}L_{jt}^{1-\theta_j}\right)^{\alpha_j} M_{jt}^{1-\alpha_j}.$$

The first order conditions for this problem are

$$W_t = \alpha_j(1-\theta_j)\frac{P_{jt}Q_{jt}}{L_{jt}} \tag{D.5}$$

$$R_{jt} = \alpha_j\theta_j\frac{P_{jt}Q_{jt}}{K_{jt}} \tag{D.6}$$

$$P_{jt}^M = (1-\alpha_j)\frac{P_{jt}Q_{jt}}{M_{jt}}. \tag{D.7}$$

The profit maximization problem of the intermediates bundling firm for sector j is given by:

$$\max_{M_{ijt}} P_{jt}^M M_{jt} - \sum_i P_{it}M_{ijt},$$

where the bundle of intermediates used by sector j, M_{jt}, is given by:

$$M_{jt} = A_{jt}^M\left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}}\right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}. \tag{D.8}$$

The first order conditions for this problem, for each sector i, are:

$$P_{it} = P_{jt}^M\left(A_{jt}^M\right)^{1-\frac{1}{\epsilon_{Mj}}}\left(\omega_{Mij}\frac{M_{jt}}{M_{ijt}}\right)^{\frac{1}{\epsilon_{Mj}}}. \tag{D.9}$$

Obtaining the equation (11) for the price of the intermediates bundle sold to sector j, P_{jt}^M, follows from solving the first order conditions for M_{ijt}, plugging into the expression for M_{jt}, and solving for P^M_{jt}.

The profit maximization problem and first order conditions for the investment bundling are symmetric and are given by:

$$\max_{X_{ijt}} P^X_{jt} X_{jt} - \sum_i P_{it} X_{ijt} \text{ s.t. } P_{it} = P^X_{jt} \left(A^X_{jt}\right)^{1-\frac{1}{\epsilon_{Xj}}} \left(\omega_{Xij} \frac{X_{jt}}{X_{ijt}}\right)^{\frac{1}{\epsilon_{Xj}}} \tag{D.10}$$

where the bundle of investment for capital specific to sector j, X_{jt}, is given by:

$$X_{jt} = A^X_{jt} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}}\right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}. \tag{D.11}$$

Similarly, equation (12) for the price of the investment bundle for sector j's capital can be obtained by solving the first order conditions for X_{ijt}, plugging into the expression for X_{jt}, and solving for P^X_{jt}.

In equilibrium, each labor, capital, intermediate bundling, and investment bundling market clears. To conserve on notation, market clearing is built into how the capital, intermediates and investment problems have been written down. With the household inelastically providing unitary labor supply each period, labor market clearing is simply given by $\sum_j L_{jt} = 1$. That leaves market clearing for final production in each sector j, which is given by:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{D.12}$$

We also note that the evolution of capital in each sector is given by the standard accumulation equation:

$$K_{jt+1} = (1-\delta_j)K_{jt} + X_{jt}. \tag{D.13}$$

For each production sector j, constant returns to scale implies:

$$W_t L_{jt} + R_{jt} K_{jt} + P^M_{jt} M_{jt} = P_{jt} Q_{jt}. \tag{D.14}$$

Therefore, the accounting definition of nominal value added is simply:

$$P^V_{jt} V_{jt} = P_{jt} Q_{jt} - P^M_{jt} M_{jt} = W_t L_t + R_{jt} K_{jt}. \tag{D.15}$$

To obtain real value added, we use a discrete-time application of the Divisia index definition, which differentiates the accounting definition of nominal value added holding prices fixed:

$$\begin{aligned} P^V_{jt} V_{jt} \Delta \ln V_{jt} &= P_{jt} Q_{jt} \Delta \ln Q_{jt} - P^M_{jt} M_{jt} \Delta \ln M_{jt} \\ \alpha_j \Delta \ln V_{jt} &= \Delta \ln Q_{jt} - (1-\alpha_j) \Delta \ln M_{jt} \\ \Delta \ln V_{jt} &= \frac{1}{\alpha_j} \Delta \ln A_{jt} + \theta_j \Delta \ln K_{jt} + (1-\theta_j) \Delta \ln L_{jt}. \end{aligned}$$

Cumulating this expression yields that real value added is given by $V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$.[46]

[46]We have normalized the implicit time-invariant constant in cumulating this expression to 1.

Finally, we can write the price index for value added in sector j, P^V_{jt}, as follows:

$$
\begin{aligned}
P^V_{jt} &= \frac{P_{jt}Q_{jt} - P^M_{jt}M_{jt}}{V_{jt}} = \frac{P_{jt}V_{jt}^{\alpha_j}\left(\left(\frac{(1-\alpha_j)P_{jt}}{P^M_{jt}}\right)^{\frac{1}{\alpha_j}} V_{jt}\right)^{1-\alpha_j} - P^M_{jt}\left(\frac{(1-\alpha_j)P_{jt}}{P^M_{jt}}\right)^{\frac{1}{\alpha_j}} V_{jt}}{V_{jt}} \\
&= P_{jt}^{\frac{1}{\alpha_j}}\left(P^M_{jt}\right)^{1-\frac{1}{\alpha_j}}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{1}{1-\alpha_j} - 1\right) = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P^M_{jt}}\right)^{\frac{1-\alpha_j}{\alpha_j}}
\end{aligned}
$$

where we use the fact that $Q_{jt} = V_{jt}^{\alpha_j} M_{jt}^{1-\alpha_j}$ and the fact that $M_{jt} = \left(\frac{(1-\alpha_j)P_{jt}}{P^M_{jt}}\right)^{\frac{1}{\alpha_j}} V_{jt}$ (from the first order conditions for intermediates, shown in equation (D.7)).

Appendix D.2. Proof of Lemma 1

We first derive expressions that relate the price of the investment and intermediates bundles to TFP growth, which we then use to construct an expression for aggregate GDP. Assumptions 1-3 imply that $P_{jt}/P_{it} = A_{it}/A_{jt}$. With this relationship, we then manipulate the expression for the price of investment (equation (12), though now common to all sectors due to Assumptions 1 and 2):

$$
\begin{aligned}
P^X_t &= \frac{1}{A^X_t}\left(\sum_k \omega_{Xi} P_{kt}^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} = P_{jt}\frac{1}{A^X_t}\left(\sum_k \omega_{Xk}\left(\frac{P_{kt}}{P_{jt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \\
&= P_{jt}\frac{1}{A^X_t}\left(\sum_k \omega_{Xk}\left(\frac{A_{jt}}{A_{kt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} = P_{jt}A_{jt}\frac{1}{A^X_t}\left(\sum_k \omega_{Xk}\left(A_{kt}\right)^{\epsilon_X - 1}\right)^{\frac{1}{1-\epsilon_X}}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
P_{jt}A_{jt} &= P^X_t A^X_t \left(\sum_k \omega_{Xk}\left(A_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X - 1}} \\
P_{jt}A_{jt} &= P^X_t B^X_t,
\end{aligned}
$$

where $B_X(t) \equiv A^X_t\left(\sum_k \omega_{Xk}A_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$. By symmetry, the following result holds for the price of intermediates, P^M_t:

$$
\begin{aligned}
P_{jt}A_{jt} &= P^M_t A^M_t \left(\sum_k \omega_{Mk}\left(A_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X - 1}} \\
&= P^M_t B^M_t.
\end{aligned}
$$

Aggregate GDP, Y_t, denoted in units of the numeraire, is given by $Y_t = \sum_i P^V_{it}V_{it}$, where V_{it} is real value added in sector i and P^V_{it} is the price of value added in sector i.

As shown in Appendix D.1, sectoral real value added and its price, can be written as:

$$V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j},$$

$$P_{jt}^V = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}} \left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M} \right)^{\frac{1-\alpha_j}{\alpha_j}}.$$

Given Assumptions 1 and 2 (implying that $\alpha_j = \alpha \ \forall j$), we can write aggregate GDP as:

$$Y_t = \sum_i P_{it}^V V_{it} = \sum_i \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{A_{it} P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \left(\frac{K_{it}}{L_{it}} \right)^{\theta} L_{it}.$$

Because of the common rental rate and wage, capital to labor ratios will be equated across sectors, and with an aggregate labor supply of 1, will simply be equal to the aggregate stock of capital, $K_t = \sum_i K_{it}$. Further, given our above derivations and our choice of aggregate investment as the numeraire, we have that $\frac{A_{it} P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} = \left(P_{it} A_{it}\right)^{\alpha} \left(B_t^M\right)^{1-\alpha} = \left(B_t^X\right)^{\alpha} \left(B_t^M\right)^{1-\alpha}$. With this, the above expression for GDP becomes:

$$Y_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}} \left(K_t\right)^{\theta} \sum_i L_{it} = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}} K_t^{\theta} = \mathcal{A}_t K_t^{\theta},$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}}$.

Appendix D.3. Proof of Proposition 1

Given the if and only if statement in the proposition, we must prove both the necessary and sufficient directions. We start with the necessary direction, showing if an ABGP exists, this requires that $\gamma^{\mathcal{A}}$ is constant and that $\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$

The requirement that $\gamma^{\mathcal{A}}$ be constant follows immediately from the aggregate production function expression from Lemma 1, $Y_t = \mathcal{A}_t K_t^{\theta}$. If Y_t and K_t grow at constant rates, that means that $\mathcal{A}_t$ must as well. Thus, the remainder of this direction of the proof entails showing that the growth rates of K_t, Y_t, W_t, X_t, and E_t^C are all equal to $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and that the growth rate of R_t is zero.

Taking the Euler equation from the household's problem (see Appendix D.1), we have that $\frac{E_{t+1}^C}{E_t^C} = \gamma_{t+1}^{E^C} = \beta \left(R_{t+1} + 1 - \delta\right)$. This implies that a constant growth rate in household expenditures implies a constant rental rate of capital, R_t, along the ABGP.

Taking the ratio of first order conditions for capital and labor in each sector, we have that $K_{jt} = \frac{\theta}{1-\theta} \frac{W_t}{R_t} L_{jt}$. Summing across sectors j, because labor is in unit aggregate supply, we can write the aggregate capital stock, K_t, as $K_t = \frac{\theta}{1-\theta} \frac{W_t}{R_t}$. Since R_t is constant along the ABGP, this implies that $\gamma^K = \gamma^W$.

Using our expressions for real sectoral value added and its price, we can rewrite the first order condition for capital in any given sector as:

$$R_t \;=\; \theta\alpha\frac{P_{jt}Y_{jt}}{K_{jt}}=\theta\alpha\frac{\frac{1}{\alpha}P^V_{jt}V_{jt}}{K_{jt}}=\theta\frac{\alpha}{1-\alpha}\left(1-\alpha\right)^{\frac{1}{\alpha}}\left(\frac{K_{jt}}{L_{jt}}\right)^{\theta-1}\left(\frac{P_{jt}A_{jt}}{(P^M_t)^{1-\alpha}}\right)^{\frac{1}{\alpha}}=\theta\mathcal{A}_tK_t^{\theta-1}.$$

Taking the ratio of this simplified first order condition for capital across time periods yields $\frac{K_{t+1}}{K_t}=\left(\frac{R_{t+1}}{R_t}\frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\right)^{\frac{1}{1-\theta}}$ and $\gamma^K=\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$, since the rental rate of capital is constant along the ABGP. Taking the ratio of the aggregate production function from Lemma 1 across time periods yields $\frac{Y_{t+1}}{Y_t}=\frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\left(\frac{K_{t+1}}{K_t}\right)^{\theta}$ and $\gamma^Y=\gamma^{\mathcal{A}}\left(\gamma^{\mathcal{A}}\right)^{\frac{\theta}{1-\theta}}=\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$, which thus implies $\gamma^Y=\gamma^K$.

Taking the capital accumulation equation, if we divide by K_t, we have $\gamma^K=(1-\delta)+\frac{X_t}{K_t}$. Since γ^K is a constant, this requires that the RHS be constant, or in other words, $\gamma^X=\gamma^K$.

The only remaining condition to verify is that aggregate consumption expenditures, E^C_t, grow at the same rate as aggregate capital. We can write GDP using expenditure side accounting as $Y_t=E^C_t+X_t$, since all these aggregates are denoted in units of the numeraire. Since we know that $\gamma^Y=\gamma^K=\gamma^X$ and γ^{E^C} is constant, then $\gamma^{E^C}=\gamma^Y=\gamma^K$. This finishes the necessity direction of the proof.

We now consider the sufficiency direction required for the proof. We now show that if $\gamma^{\mathcal{A}}$ is constant, then an ABGP exists. We do this by construction. We set $\gamma^K=\gamma^X=\gamma^Y=\gamma^{E^C}=\gamma^W=\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and we set R_t to be a constant such that the Euler Equation holds: $R_{t+1}=\frac{1}{\beta}\gamma^{E^C}-(1-\delta)$. Given our assumption that $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}>\beta(1-\delta)$, this will produce a non-negative rental rate for capital.

Given an initial value of $\mathcal{A}_t$, this value of R implies a unique value for K_0 from the rewritten first order conditions. It is then straightforward to construct X_0 to satisfy capital accumulation, given K_0 and γ^K. Finally, we can determine the initial condition for expenditures, using the expenditure side accounting relationship, with $E^C_0=Y_0-X_0=\mathcal{A}_0K_0^{\theta}-K_0\left(\gamma^K-(1-\delta)\right)$.

Lastly, to show that transversality holds, we need that $\lim_{t\to\infty}\beta^t\frac{K_t}{E^C_t}=0$. Given that we have constructed the path such that $\gamma^K=\gamma^{E^C}$,$\frac{K_t}{E^C_t}$ will be a constant along this path and thus the limit will be satisfied. This completes the proof in the sufficiency direction.

Appendix D.4. Proof of Lemma 2

We prove the result for intermediates and ϵ_M first; the result for investment with ϵ_X follows by the symmetry of the CES functions. For ease of exposition of the proof, we define $g^M(\epsilon_M)$ as follows:

$$g^M(\epsilon_M)=\left(\sum_i s^M_{it-1}(\gamma^A_{it})^{\epsilon_M-1}\right)^{\frac{1}{\epsilon_M-1}}.$$

Thus, the objective is to show that $g^M(\epsilon_M)$ is weakly increasing in ϵ_M. For ease of exposition, we also suppress the A superscript on γ^A_{it} and define $\gamma_i\equiv\gamma_{it}$.

$g^M(\epsilon_M)$ depends on ϵ_M in two ways—both directly, as an exponent on γ_{it}^A and in the exponent for the overall sum, but also indirectly, through its impact on s_{it-1}^M, which is itself a function of ϵ_M. With assumptions 1-3, s_{it-1}^M can be written as a function of exogenous values, including ϵ_M:

$$s_{it-1}^M(\epsilon_M) = \omega_{Mi} \frac{A_{it-1}^{\epsilon_M - 1}}{\sum_j^N \omega_{Mj} A_{jt-1}^{\epsilon_M - 1}}.$$

Our goal is to show that for any $\epsilon_1 > \epsilon_2$, $g^M(\epsilon_1) \geq g^M(\epsilon_2)$. We show this in two steps. First, we define the function $\tilde{g}(\sigma, \epsilon_M) = \left(\sum_i s_{it-1}^M(\sigma)\gamma_{it}^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}}$. We first show that for fixed σ and $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. The second step defines the function $\hat{g}(\epsilon_M, \sigma) = \left(\sum_i s_{it-1}^M(\epsilon_M)\gamma_{it}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ and shows that $\hat{g}(\epsilon_1, \sigma) \geq \hat{g}(\epsilon_2, \sigma)$. Then, given these two substeps, the final result follows from the following sequence of inequalities:

$$g(\epsilon_1) = \tilde{g}(\epsilon_1, \epsilon_1) \geq \tilde{g}(\epsilon_1, \epsilon_2) = \hat{g}(\epsilon_1, \epsilon_2) \geq \hat{g}(\epsilon_2, \epsilon_2) = g(\epsilon_2). \tag{D.16}$$

We note that the lemma makes the assumption of positive dependence in the form of $\mathbb{E}\left[ln(A_{it}) \mid \gamma_{it}^A = a\right]$ being weakly increasing in a. This assumption is not needed until Step 2, and so we demonstrate Step 1 for the more general case without this assumption.

Step 1: For $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. This first step of the proof follows from an application of Jensen's inequality. Jensen's inequality tells us that for any convex function, $\phi(x)$, any real valued function $h(x)$, and any set of non-negative weights a_i with $\sum_i a_i = 1$, $\sum_i a_i \phi(h(x_i)) \geq \phi(\sum_i a_i h(x_i))$. The inequality is reversed in the case where $\phi(x)$ is concave.

First, begin with the case where $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $\phi(x) = x^{\frac{\epsilon_1 - 1}{\epsilon_2 - 1}}$. This function is convex because $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $h(x) = x^{\epsilon_2 - 1}$ and $a_i = s_{it-1}^M(\sigma)$. Jensen's inequality thus implies the following result:

$$\left(\sum_i s_{it-1}^M(\sigma)\gamma_{it}^{\epsilon_2 - 1}\right)^{\frac{\epsilon_1 - 1}{\epsilon_2 - 1}} = \tilde{g}(\sigma, \epsilon_2)^{\epsilon_1 - 1} \leq \left(\sum_i s_{it-1}^M(\sigma)\gamma_{it}^{\epsilon_1 - 1}\right) = \tilde{g}(\sigma, \epsilon_1)^{\epsilon_1 - 1}.$$

Exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1 - 1}$, which is a positive exponent, completes the result for this case.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 < \frac{\epsilon_1 - 1}{\epsilon_2 - 1} < 1$, then it must be that $\epsilon_1 < 1$. In this case, $\phi(x)$ is now concave, which reverses the above inequality. However, because $\epsilon_1 < 1$, the step of exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1 - 1}$ again reverses the inequality and ensures the result holds.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 > \frac{\epsilon_1 - 1}{\epsilon_2 - 1}$, then $\epsilon_2 < 1$ and $\epsilon_1 > 1$, and $\phi(x)$ is again convex and $\frac{1}{\epsilon_1 - 1}$

is a positive exponent, so the result still holds.

Finally, consider the case where either $\epsilon_1 = 1$ or $\epsilon_2 = 1$. Although $\tilde{g}(\epsilon_M)$ is undefined in this case, we consider instead the limiting result, defining $\tilde{g}(\sigma, 0) = \prod_i \gamma_{it}^{s_{it-1}^M(\sigma)}$. Here we apply Jensen's inequality using $\phi(x) = \ln(x)$ and $h(x) = x^{\epsilon_1 - 1}$. If $\epsilon_2 = 1$ and $\epsilon_1 > 1$, then we have that:

$$(\epsilon_1 - 1)\ln(\tilde{g}(\sigma, \epsilon_1)) = \ln(\sum_i s_{it-1}^M(\sigma)\gamma_{it}^{\epsilon_1 - 1}) \geq (\epsilon_1 - 1)\sum_i s_{it-1}^M(\sigma)\ln(\gamma_{it}) = (\epsilon_1 - 1)\ln(g(\sigma, 0)).$$

Dividing both sides by $\epsilon_1 - 1$ and exponentiating both sides of the inequality yields the result. In the case where $\epsilon_1 = 1$ and $\epsilon_2 < 1$, the same steps can be followed, replacing ϵ_1 with ϵ_2, but now since $\epsilon_2 < 1$, the final step of dividing both sides by $\epsilon_2 - 1$ will reverse the inequality, proving the result.

Step 2: For $\epsilon_1 > \epsilon_2$, $\hat{g}(\epsilon_1, \sigma) \geq \hat{g}(\epsilon_2, \sigma)$. To prove this inequality, we show that $\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} \geq 0$.

Taking the partial derivative, we obtain the following result:

$$\begin{aligned}\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} &= \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma}\sum_i \frac{\partial s_{it-1}^M(\epsilon_M)}{\partial \epsilon_M}\gamma_{it}^{\sigma - 1} = \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma}\left(\sum_i s_{it-1}^M \gamma_{it}^{\sigma-1}\ln(A_{it-1})\right.\\ &\left. - \left(\sum_i s_{it-1}^M \gamma_{it}^{\sigma-1}\right)\left(\sum_i s_{it-1}^M \ln(A_{it-1})\right)\right) = \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma}\text{Cov}\left(\ln(A_{it-1}), \gamma_{it}^{\sigma-1}\right).\end{aligned}$$

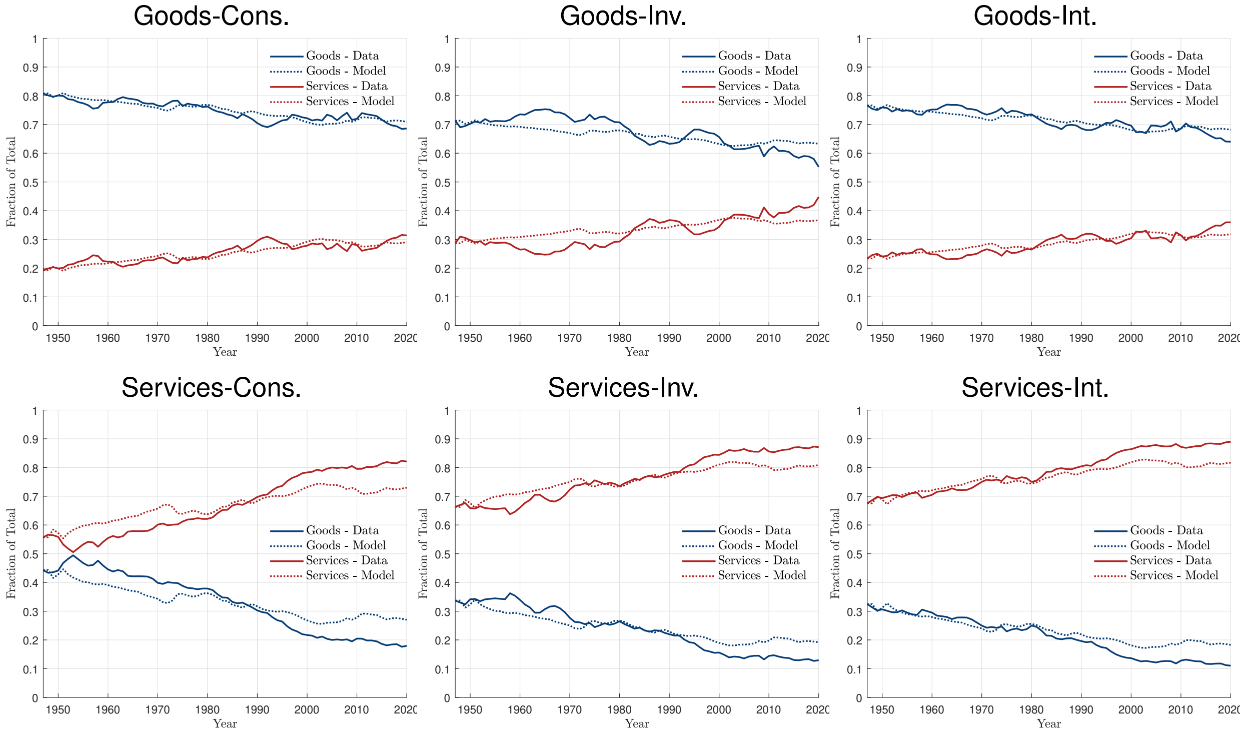
where $\text{Cov}(\ln(A_{it}), \gamma_{it}^{\sigma-1})$ is the covariance between $\ln(A_{it-1})$ and $\gamma_{it}^{\sigma-1}$ where probability weights across sectors are defined by the shares s_{it-1}^M. Given the weak positive dependence assumption, that $\mathbb{E}\left[ln(A_{it-1}) \mid \gamma_{it}^A = a\right]$ is weakly increasing in a, the sign of this covariance term will be the sign of $\sigma - 1$.[47] Thus, since this covariance has the same sign as $\sigma - 1$, the result will go through. Since we know that $\hat{g}(\epsilon_M, \sigma)^{2-\sigma} > 0$ and the entire expression is multiplied by $\frac{1}{\sigma - 1}$, this ensures that $\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} \geq 0$. This completes step 2 of the proof.

Given the successful completion of steps 1 and 2, the proof for intermediates follows from the inequalities in equation (D.16) and the proof for investment follows by symmetry.

Appendix E. Additional Calibration and Growth Accounting Results

[47] This can be seen by an iterated expectations argument. Consider two random variables X and Y which have, without loss of generality, zero mean. The covariance of X and Y is $COV(X, Y) = \mathbb{E}[XY] = COV(X, \mathbb{E}[Y \mid X])$. If $\mathbb{E}[Y \mid X = x]$ is increasing in x, then, since the covariance of two increasing functions of X is positive, then the sign of the covariance term in our case will depend on the sign of $\sigma - 1 < 0$.

Figure E.7: Model Calibration Fit to Structural Change Patterns in Intermediates, 6 Sector Level, 1947-2020



Notes: Each panel plots the fraction of intermediates purchased from goods (blue lines) and services (red lines). Data series are solid lines; model series are dashed lines.

Appendix E.1. Additional Calibration Results

Here, we report the model's fit to intermediate expenditure and investment expenditure patterns in all six sectors and how the calibration compares to the shift-share decomposition exercises described in Section 2 and presented in Appendix B.

Figures E.7 and E.8 present the model fit to the share of intermediates and investment purchased from goods and services sectors in each of our six sectors. The model fits structural change in intermediates better in the three goods sectors, especially before 2008; the model series account for nearly 90% of the structural change in intermediates for goods sectors through 2008, compared to 70% for services sectors. However, the decline in fit post-2008 is primarily concentrated among goods sectors. In contrast, the fit to investment structural change is generally better for services sectors, with some overshooting in overall structural change in investment among goods sectors.

In Tables E.1 and E.2, we present a shift-share decomposition of our model's results for struc-

Figure E.8: Model Calibration Fit to Structural Change Patterns in Investment, 6 Sector Level, 1947-2020

Notes: Each panel plots the fraction of investment purchased from goods (blue lines) and services (red lines). Data series are solid lines; model series are dashed lines.

Table E.1: Shift-Share Decomposition of Services Share of Production of Intermediates, Model vs. Data

				Decomposition	
	1947	2019	Δ	within	between
Data	0.35	0.68	0.32	0.18	0.14
				(57%)	(43%)
Model	0.36	0.56	0.21	0.12	0.09
				(56%)	(44%)

Notes: The table reports the results of a shift-share decomposition of the share of services production of intermediates over time.

Table E.2: Shift-Share Decomposition of Services Share of Production of Investment, Model vs. Data

				Decomposition	
	1947	2019	Δ	within	between
Data	0.20	0.40	0.20	0.21	-0.01
				(104%)	(-4%)
Model	0.20	0.45	0.25	0.27	-0.02
				(109%)	(-9%)

Notes: The table reports the results of a shift-share decomposition of the share of services production of investment over time.

Table E.3: Growth Decomposition by Decade, 1950-2020

Sources	1950s Δ	1950s %	1960s Δ	1960s %	1970s Δ	1970s %	1980s Δ	1980s %	1990s Δ	1990s %	2000s Δ	2000s %	2010s Δ	2010s %
All	0.16	100	0.18	100	0.11	100	0.14	100	0.19	100	0.18	100	0.04	100
Consumption-Specific	0.07	44	0.03	17	0.08	74	0.00	3	0.03	18	0.09	50	0.01	36
Investment-Specific	0.04	25	0.08	45	0.03	29	0.05	37	0.09	47	0.10	58	0.07	166
Intermediates-Specific	0.05	30	0.07	37	0.00	3	0.08	60	0.07	39	0.02	9	-0.03	-63
Bundling	-0.01	-4	0.00	0	0.00	2	-0.00	-3	-0.02	-9	0.00	2	0.01	36

Notes: The Table reports decade-by-decade version of Tables 2.

tural change in intermediates and investment. Although the model does not replicate the entire rise in the share of intermediates produced by services, it does closely match the composition of changes over time due to within and between sector forces.[48] The model also replicates the result that all of structural change in investment is occurring within sectors.

Appendix E.2. Additional Growth Accounting Results

In this section, we present three additional results. First, we report an extension of Table 2 where we report the composition of aggregate GDP growth decade by decade, further highlighting the rising importance of investment-specific technical change over time. Second, we report separately the contributions of investment-bundling and intermediates bundling technical change to the growth accounting decomposition in Table Table 2. Third, we report the growth accounting decomposition results from the body of the paper where we extend the model to include a non-homotheticity in preferences using a Stone-Geary specification.

Table E.3 shows decade by decade log changes in aggregate GDP per worker and in the same sets of counterfactuals presented in Table 2— consumption-specific technical change, investment-specific technical change, intermediates-specific technical change, and only bundling technical change. We observe a clear upward trend in the importance of investment-specific technical change over time, explaining more than 100% of all growth between 2010-2019.

Table E.4 reports the growth accounting decomposition from Table 2 with two additional counterfactuals – aggregate GDP per worker growth with only TFP growth in investment-bundling or

[48]When comparing the model results to the data, we aggregate intermediate expenditures in each sector slightly differently than in the empirical results presented in Appendix B. Because the share of intermediate spending in gross output is constant over time, we aggregate intermediate expenditures in the data using modified gross output weights, defined by gross output in each sector multiplied by its average ratio of intermediates spending to gross output; this accounts for the fact that the between sector contribution falls from 47% in Table B.2 to 43% in Table E.1.

intermediates-bundling TFP. Investment-bundling TFP contributes negatively to GDP per worker growth and intermediates-bundling TFP contributes positively, but these contributions are generally modest. The one exception is that in the final years of the sample, with aggregate growth significantly slower, the contributions of these bundling TFP terms are larger overall. However, they still contribute a clear minority to overall economic growth.

Table E.4: GDP per Worker Growth Decomposition, Details of Bundling Technical Change, 1947-2019

	Aggregate GDP per Worker Growth: $\Delta \ln(Y_t)$							
	1947-2019		1960-1980		1980-2000		2000-2019	
All TFP	1.04	100	0.29	100	0.33	100	0.22	100
Investment-Bundling	-0.08	-7	-0.02	-7	-0.03	-8	-0.03	-14
Intermediates-Bundling	0.07	7	0.02	7	0.01	2	0.05	23

Notes: The table shows long-run log changes in aggregate GDP per worker across different periods for three alternative simulations: (1) the full model simulation with all measured TFP series and two counterfactual simulations with TFP growth only from investment-bundling TFP and intermediates-bundling TFP. Counterfactual changes are expressed as a percent of the change from the full model.

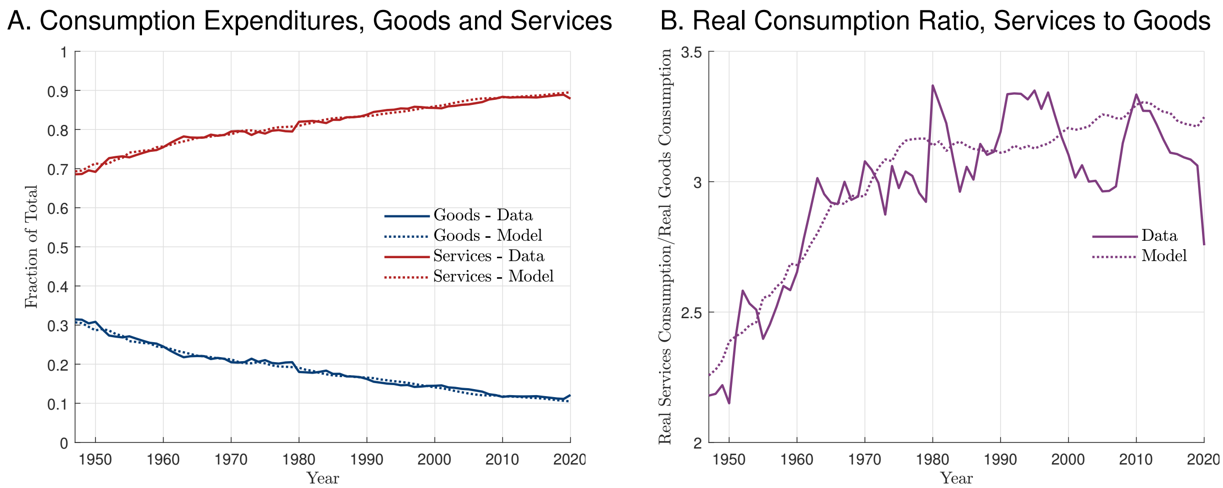
Finally, we consider an extension to the model where the per-period utility function (presented under our 6 sector calibration) is given by:

$$U(C_{gt}, C_{st}) = \ln\left(\left[\omega_{Cg}^{1/\epsilon_C} C_{gt}^{\frac{\epsilon_C - 1}{\epsilon_C}} + (1 - \omega_{Cg})^{1/\epsilon_C} \left(C_{st} + \bar{c}_s\right)^{\frac{\epsilon_C - 1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C - 1}}\right), \tag{E.1}$$

where $\bar{c}_s$ is a constant non-homotheticity term for services consumption. The introduction of this term implies the presence of income effects for structural change, as the household endogenously chooses to spend a larger fraction of consumption on services as incomes rise. The introduction of this term (in a form akin to Stone-Geary preferences) provides an additional means by which to fit structural change in consumption and may potentially impact our growth accounting results, including the impact of consumption structural change on overall economic growth.

With the introduction of $\bar{c}_s$, we now calibrate the parameters of the per-period utility function along the transition path of the model. To calibrate ϵ_C, $\omega_C g$, and $\bar{c}_s$, we choose the parameters that minimize the squared distance between the model and data series for: (1) annual share of consumption expenditures produced by the goods sector, 1947-2020, (2) annual ratio of the quantity of services consumption to goods consumption, 1947-2020, and (3) the initial share of consumption

Figure E.9: Calibration Fit with Non-Homotheticity, 1947-2020



Notes: Panel A shows the time series of the share of consumption spending produced by goods and services sectors in the model (dashed lines) and in the data (solid lines); panel B shows real consumption of services divided by real consumption of goods in the model (dashed line) and the data (solid line).

expenditures produced by the goods sector (1947).[49] Figure E.9 shows the fit of the calibration to these moments along the transition path; the model now matches structural change in consumption expenditures almost exactly along the entire transition path and also provides a very good fit to the quantity ratio of services consumption to goods consumption over time. The parameter values that generate this fit are $\epsilon_C = 0.32$, $\omega_{Cg} = 0.12$, and $\bar{c}_s = 0.61$. Thus, while goods and services are still strong complements in overall consumption, the degree of complementarity less than in our baseline calibration, where the best fit to consumption structural change was given by $\epsilon_C = 0$.

Tables E.5 and E.6 then reproduce GDP per worker growth accounting decompositions reported in Tables 2 and 3 in the body of the paper for this extension with modified preferences. The results of Table E.5, reporting the decomposition of aggregate GDP per worker growth into consumption-, investment-, and intermediates-specific technical change are almost identical to our baseline findings in Table 2, with only minor differences. The most notable differences with the introduction of the non-homotheticity in preferences is visible in the Cobb-Douglas counterfactuals in Table E.6 in the case of consumption. Because goods and services are still complements in consumption, eliminating the portion structural change in consumption due to relative prices implies faster overall

[49]The real quantity of services consumption and goods consumption are obtained using our data on consumption expenditures by producing sector and the price of consumption produced by goods and services.

Table E.5: GDP Growth Decomposition with Non-Homotheticity, 1947-2019

	Aggregate GDP per Worker Growth: $\Delta \ln(Y_t)$							
	1947-2019		1960-1980		1980-2000		2000-2019	
All	1.04	100	0.29	100	0.33	100	0.22	100
Investment-Specific	0.48	46	0.11	39	0.15	46	0.17	79
Intermediates-Specific	0.27	26	0.08	26	0.16	49	-0.01	-3
Consumption-Specific	0.35	34	0.11	37	0.03	10	0.10	45
Bundling	-0.00	0	-0.01	-3	-0.02	-8	0.03	12

Notes: The table shows long-run log changes in aggregate GDP per worker across different periods for five alternative simulations (all with a non-homotheticity in preferences): (1) the full model simulation with all measured TFP series and three counterfactual simulations with TFP growth only from (2) investment producers and exogenous investment-bundling TFP ("investment-specific technical change"), (3) intermediates producers and exogenous intermediates-bundling TFP ("intermediates-specific technical change"), (4) consumption producers ("consumption-specific technical change"), (5) and both exogenous investment-bundling TFP and intermediates-bundling TFP. Counterfactual changes are expressed as a percent of the change from the full model; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. Furthermore, the bundling TFP series are already included in the investment- and intermediates-specific technical change counterfactuals, so adding their percentage contribution will overstate total growth.

Table E.6: GDP Growth, Cobb-Douglas Counterfactuals with Non-Homotheticity, 1947-2019

	GDP Growth: $\Delta \ln Y_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Baseline	1.04	100	0.29	100	0.33	100	0.22	100
Consumption Cobb-Douglas	1.07	103	0.29	101	0.34	103	0.23	108
Investment Cobb-Douglas	0.99	95	0.29	99	0.32	98	0.17	80
Intermediates Cobb-Douglas	1.08	104	0.30	103	0.36	110	0.22	100

Notes: The table reports log changes in aggregate GDP per worker, $\ln(Y_t)$, across different periods under different assumptions regarding CES bundling technologies within our extended model with a non-homotheticity in preferences. The table reports counterfactuals for the cases where consumption, investment, or intermediates aggregation is Cobb-Douglas, ruling out structural change in that commodity. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

economic growth. However, because the degree of complementarity is smaller with the introduction of income effects, the magnitude of the growth drag due to consumption complementarity is roughly 40-45% less than in our baseline results in Table 3.